Comprehending Algebra Word Problems in the First and Second Languages

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1. Introduction

Learning school subjects in a language that is not the students' first language (L1) has become a reality in many parts of the world due to sociopolitical reasons, advances in technology and better and faster communication systems. Understanding the psychological dimensions of bilingualism within the context of the classroom is crucial for educational policy makers who make decisions about curriculum development, instructional practices, and student evaluation. From a cognitive perspective, examining the processes of comprehension and understanding of academic subjects learned in a second language (L2) is pivotal to addressing issues of learning and instruction. The impact of bilingual education on the teaching and learning process is complex, especially since one goal is to make the process of teaching and learning in a second language as natural as in one's first language, without compromising the knowledge, competence and performance of either language.

The present study was designed to address some of the questions involved in the area of learning and understanding academic subjects in the L2. It examined a specific area of study within a cognitive framework, namely students' comprehension as they read and solved algebra word problems in their L1 (Spanish) and L2 (English). The study was designed in such a way that it allowed for a crosslinguistic comparison of text understanding in both languages, a situation the students that participated in the study are faced with every day. Thus, they would have to be sensitive to textual features and differences in both languages.

Word problems have traditionally been difficult for many people. In many instances, individuals who seem to lack adequate computational skills in solving word problems demonstrate these skills when problems are presented in numerical form. Previous research from a discourse perspective, has shown that most of the difficulty with word problems arises from a mismatch between text comprehension, situation comprehension, and problem solving procedures. This difficulty is compunded by the specialized nature of the problems' text.

A variety of explanations coming from research in elementary arithmetic has been offered to account for how the wording of arithmetic story-problems influences performance. Some researchers have focused on the semantic structure of addition and subtraction word problems (Carpenter & Moser, 1982, 1983; Riley & Greeno, 1988; Riley, Greeno, & Heller, 1983). Other researchers have emphasized the mathematics basic ability or knowledge required to represent the semantic relations (Gelman & Greeno, 1989; Resnick, 1989; Riley & Greeno, 1988). Others offer linguistic explanations (Cummins, 1991; Cummins, Kintsch, Reusser, & Weimer, 1988; Hudson, 1983) in which the difficulty is attributed to comprehension aspects that have to do with language ambiguity and the abstract nature of mathematical language. Although some authors have suggested that syntactic simplification may help children's understanding of word problems (De Corte, Verschaffel, & De Win., 1985), syntactic details seem to be of little use to students for distinguishing situations presented in word problems (Marshall, 1995). Nevertheless, Cummins et al. (1988) found that children miscomprehended more word problems that contained ambiguous and abstract language than problems worded in simpler language terms. They also found that solution errors were related to the ability to recall the statement of the problem correctly. In other words, their study suggests that appropriate representation of word problems is highly language and textbase dependent. Similarly, Kintsch and Greeno (1985) also argued that the root of arithmetic word problem difficulty can be traced back to text comprehension. They believed that the nature of comprehension itself is determined by the purposes for which the text is read. Thus, there are textual aspects that warrant the use of the specific knowledge structures and

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operations required to arrive at the correct solution. In her socio-linguistic view of L2 reading, Bernhardt (1991) claimed that the pre-set nature of reading is determined by the nature of texts and the learned ways of dealing with such texts in the L1 context. In the case of word problems, the text is already pre-set for mathematical understanding. That is, the students must approach this type of text within the mathematical framework. They must know that the words have mathematical meaning, regardless of the 'real world' situation presented in the text.

The difficulty of algebra word problems has also been recognized and researched since the 1960s with the work of Paige and Simon (1966). More recently, Nathan, Kintsch and Young (1992) proposed a model for algebra word problems that includes three mutually constraining levels of representations: the propositional textbase, the situation model, and the problem model. They argue that the task of solving word problem relies on students' reading comprehension abilities, so that faulty solutions to word problems may be traced to incorrect text comprehension and the inability to access relevant background knowledge.

It is widely accepted that students’ linguistic abilities play an important role in their learning and conceptual processing of academic subjects. This role is particularly important in the domain of mathematics, where students have to use their linguistic abilities before conceptualizing a problem in mathematical terms, so that they can arrive at a correct numerical representation and problem solution. In this sense, achievement in mathematics is influenced by the student's proficiency in the language of instruction (Zepp, 1981). Miura (2001) argued that students' mathematics understanding is influenced by the cultural factors that shape the representations that are found within the classroom. These factors include the characteristics of the language of instruction, and the characteristics of the mathematical terms that are specific to that language. The representations are of two types: instructional, which refer to classroom discourse, (i.e., students teacher exchanges and the language of instruction) and cognitive, which refer to the individual student's representations.

Any mathematics curriculum includes four basic areas, namely concepts, computation, applications, and problem solving. Mathematical concept formation is a complex process that involves perceiving the underlying relationships between mathematical ideas (Reed, 1984). To achieve this, the student must have the ability to translate from the verbal formulation to the underlying mathematical relationship (Jones, 1982). Then, the student must also have the ability to construct the mathematical conceptual representation on which the problem solving process will operate (Nesher & Teubal, 1975). During these activities, students may encounter difficulty in understanding the language of the teacher and or the text. Kane (1968; 1970) argued that mathematical English and ordinary English are so different that readers require different skills and knowledge to achieve sufficient levels of reading comprehension. To attain this goal students must be equipped with the linguistic skills that allow them to learn to understand the highly specialized and contextualized language of mathematics. Fillmore and Snow (2000) have argued that researchers and practitioners do not truly know what are the specific language skills that are required for academic English as compared to standard English skills.

Mathematical language has its own vocabulary, syntax, semantic and discourse properties. In terms of vocabulary, mathematics includes words that are specific to the domain (e.g., coefficient, denominator, etc.), and day to day words that take on a specific meaning within the context of mathematics (e.g., equal, rational, table, column, etc.). Competence in this specific vocabulary is crucial to students' mathematical understanding especially as they progress to higher grades. It means that they have to learn that the terms are related to the context in which they occur. Additionally, students have to learn another vocabulary that includes the extensive system of mathematical symbols, which have their own meaning within the mathematical context. These symbols increase in conceptual density as students move up in the curriculum. Students must also understand that there is a lack of a one to one correspondence between mathematical symbols and the words they represent, and the role of logical connectors (e.g., if...then, if and only if, but, etc.) which indicate the nature of the relationship between parts of the text (Crandall, Dale, Rhodes, & Spanos, 1987; Kessler, Quinn, & Hayes, 1985).

Learning to communicate verbally and in writing about mathematics is important. This can be emphasized in the classroom by addressing correct use of vocabulary and grammar, encouraging the use of both written and spoken mathematical language, assisting with the translation of English phrases or sentences to mathematical language and, in general, encouraging students to discuss mathematics (Oldfield, 1996). Friedlander and Tabach (2001) believe that verbal reasoning can be useful in solving problems, although its use in the classroom has not been fully legitimized. However, they also caution that, despite allowing students to make the connection of mathematics with other domains and everyday life, verbal representation can also interfere with mathematics communication given its ambiguous, distracting, and misleading nature, its lack of universality and its reliance on personal style.

Knowing how to manipulate the vocabulary and syntax of the mathematical language is directly related to the ability to correctly infer the meaning from the language of text. Students generally engage in three types of activities when performing mathematical activities: they must understand the language of the problem or the text; they must formulate the mathematical concept(s) required; and they must translate the concepts into mathematical symbols which they can manipulate for computational purposes. Thus, correct problem solving performance in algebra word problems can often be achieved through the identification of key words in the text and the words that link them (i.e., their referents). However, there is evidence that the use of key words to arrive at the proper representation and solution of problems can be counterproductive and may stay with the students as an incorrect strategy that may backfire in higher grades (Marshall, 1995). Problems that use the same words may require different solutions. In addition, the strategy may be an obstacle for students' own problem solving strategies to emerge (Carey, Fennema, Carpenter & Franke, 1995). Appropriate use of key words depends on the student's knowledge of how reference is indicated in the problem. In algebra equations students must be able to identify the variables' referents to correctly translate the words of a problem into the corresponding equation symbols. They have to understand that variables stand for numbers and not things or persons (Clement, 1981; Mestre, Gerace, & Lockhead, 1982). Misunderstandings in this area can lead to the typical 'reversal errors' committed when students who are reading word problems in algebra reverse the variable to be found. As a result, these students will place numbers and or variables in the wrong place in the equation. This type of error is apparently due to the students' expectation of a one to one correspondence usually found in word problems at the beginning levels of mathematics learning. Clement (1982) argued that reversal errors appeared to occur also with other types of representations, such as tables, figures, pictures, etc., where students are also required to write an equation. Sebrecht, Enright, Bennett, and Martin (1996) argued that these types of errors can be explained by the effect of "overall linguistic forms of the problem statement rather than the component linguistic attributes" (p.313). What they refer to as the overall linguistic forms pertains to the way language is used in the problem statement, whereas what they refer to as the component linguistic attributes pertains to semantic representation, that is, deeper representation of the problem statement (e.g., terms that are used to specify relations, and types of propositions). Sebrecht et al. did not find the latter to be good predictors for problem difficulty.

Dale and Cuevas (1987) believe that language and mathematics may be most intricately related at the cognitive and metacognitive levels. They also believe, as do other authors (Aleve & Koedinger, 2002; Garofalo & Lester, 1985; Kessler et al., 1985) that metacognition and mathematical thinking are two key factors that affect the outcome of cognitive activities in mathematics.

These issues are important for L2 students who have the ability to think mathematically in their L1, but may lack the language ability to deal with mathematical concepts in their L2, or vice versa.

1.1 The role of second language

Understanding the relationship between language, discourse and mathematics is particularly important in the case of students learning high school subjects in their L2, since the L2 has to be included in the relation. For instance, the background knowledge needed to work on word problems entails not only mathematics but also L2 knowledge. L2 knowledge, in turn, includes reading ability in L2 which is a task in and of itself. There is evidence that reading in one's L2 is often more difficult and slower than in one's first language, even for fluent bilinguals (Alderson, 1984; Favreau & Segalowitz, 1982; Segalowitz, 1986). When reading in the L2, the reader must be able to recognize the lexical and syntactic structures of the L2, map them into their underlying semantic structures, and integrate them with the entire discourse system (Donin & Silva, 1994). In doing so, the L2 reader might use a combination of reading strategies that include those that are typical of native speakers of the target language and L1 reading strategies. In a study investigating reading and comprehension strategies of Spanish bilingual university students when reading syntactically ambiguous sentences in both languages, Berdugo (1991) found that in general, the reading times for this bilingual group were similar to those of a group of unilingual Spanish readers. However, the comprehension results for the bilingual group were similar to a monolingual English group. Specifically, the English and bilingual groups made significantly fewer attachments to a verb phrase than did the Spanish group (Berdugo & Hoover, 1997). These results point to the relevance of the L2 knowledge bilingual readers must bring to the task, which might impact upon their performance. On the other hand, Chen and Donin (1992) found that domain relevant knowledge of texts had a stronger effect than L2 proficiency in a comprehension task for Chinese university students studying engineering and biology in their L2. Both L2 proficiency and domain relevant knowledge affected reading time in English.

An important issue raised by Bernhardt (1991), which is key to the present study, is the contextual nature of L2. Bernhardt offered a sociocognitive view of L2 reading, in which text features such as linguistics (syntax and semantics), pragmatics, intentionality, content and topic interact with the reader. She pointed out that L2 readers approach the text from their L1 framework, which is not the same framework as that of the audience for whom the text is intended.

The notion of L2 within a meaningful context has also been discussed within the area of L2 proficiency. Snow (1987) identified at least two different dimensions of language proficiency in bilingual children: contextualized and decontextualized language skills. Since these skills are independent, facility in interpersonal language use may not imply the ability to use the language in academic situations. Snow’s view is compatible with Cummins' (1983) criticisms of assessment practices that assume that the language proficiency required for L2 face to face communication is no different from that required for performance on a L2 cognitive/academic task, an assumption which has led to the conclusion that poor performance on a L2 verbal IQ test is a function of deficient cognitive abilities. Researchers working in the area of L2 proficiency propose a framework that could address the question of what constitutes 'language proficiency' and its implications for bilingual education, language pedagogy, and testing. This framework is based on the notion that language proficiency is contextualized along two continua: one related to the range of contextual support available for expressing or receiving meaning and the second related to the developmental aspects of communicative proficiency in terms of the degree of active cognitive involvement in a task or activity.

Finally, in terms of the cognitive benefits of being bilingual, Secada (1991a &b, 1995) argued that his research with bilingual children doing arithmetic word problems revealed that bilingualism might provide cognitive benefits to students solving arithmetic word problems in both languages (Spanish and English) that, according to conventional wisdom they should not have been able to solve in early arithmetic. Moreover, the cognitive benefits of being bilingual are evident, at least, initially in academic subjects, and that these benefits depend on the development of decontextualized academic language proficiency. Similarly, Bialystok (1991) suggested that L2 literacy may support the development of children’s control and analytical capacities over language and thought in general, producing awareness and skills that may be lacking in children who have only developed L1 literacy. This suggestion is supported by Adetula (1989) who found that schooled Nigerian children performed similarly to older and unschooled children in their two languages, Yoruba and English.

These views about the cognitive benefits of being bilingual are consistent with those of researchers who investigate metalinguistic awareness as part of language competence. There is evidence of its role as a reliable predictor of the acquisition of L2 competence. Furthermore, it has been shown that language aptitude is significantly related to metacognitive awareness (Bialystok & Frohlich, 1978; Masny & d'Anglejan, 1985; Schachter, Tyson, & Diffley, 1976). Studies have also shown that bilingual children demonstrate greater metalinguistic awareness than unilingual children (Ben Zeev, 1977). Hakuta (1984) found similar results with children with low levels of proficiency. In general, research has found higher measures of cognitive functioning (measures have included cognitive flexibility, metalinguistic awareness, concept formation, and creativity) and academic achievements as well as the development of positive cross-cultural attitudes in children and adults who have received bilingual instruction since their elementary grades (Cumming, 1990; Diaz, 1983; Met, 1991; Ringbom, 1987).

1.2 Content based L2 instruction

Content based methods of instruction are based on the assumption that L2 learning should take place incidentally in a meaningful context. Therefore, the content areas of the regular curriculum are taught in the foreign language, so that language development is integrated with content development. These methods are believed to provide a framework for developing higher-level cognitive skills such as critical thinking (Curtain & Martinez, 1990). The best examples of content based instruction are immersion programs and Foreign Language in the Elementary School (FLES) programs. These two approaches to foreign language instruction have been criticized for not fully addressing the issue of grammar instruction. Supporters of the early immersion and FLES programs have argued that young children lack the cognitive maturity to deal with abstract syntactic rules. Furthermore, the FLES programs have the added problem of a mismatch between the emphasis at the elementary level on vocabulary development, and on grammar at the secondary level (Met, 1991). Consequently, although learners may acquire native-like proficiency in their receptive skills (i.e., listening and reading), they rarely achieve this level of proficiency in their production skills (i.e., speaking and writing).

Cohen (1994) has stated that without intervention measures "immersion students tend to fossilize their language ability at a level which is adequate for the immersion classroom but which is not native-like" (p. 173). These criticisms have been addressed in immersion studies that support the view of context-based L2 instruction that fosters the integration of experiential (i.e., L2 learning embedded in content) and analytic (i.e., L2 learning based on specific features of the language such as grammar, phonology, etc.) teaching strategies that avoid learning those specific language features in isolation. Instead, communicative language characteristics, along with functional and structural ones are emphasized (Day & Shapson, 1991; Lyster, 1990; 1994). In a study that sought evidence for the above view of L2 learning, Lyster (1994) found that English students in a French immersion program developed more appropriate use of French forms (i.e., use of the informal *tu* vs the formal *vous*), that would otherwise be considered fossilized in the immersion interlanguage that usually develops by the eighth grade (Lyster, 1987). His views fall within information-processing approaches to L2 learning where control processes are seen as being activated to restructure the L2 knowledge representation. Tyler (2001) shares similar views with respect to using techniques that improve listening skills in a foreign language that not only would represent a cognitive load in working memory, but that non native speakers may tend not to overcome. He believes that students' poor aural language processing skills may be concealed by the knowledge of the topic they are listening to. He found that experienced non native English speakers compensated for poor low level listening comprehension abilities with topic knowledge. Moreover, the non native listeners appear to make more use of working memory resources than natives when topic was not available. He argued that this form of compensatory behavior is an obstacle to the development of automatic processes in language processing from controlled ones.

As L2 researchers and practitioners continue to be concerned with the development of proper levels of language proficiency within content based instruction, they are also concerned that attention to language issues not be detrimental to the teaching and learning of academic subjects. In a study that investigated this issue, Pepin and Dionne (1997) found appropriate levels of performance and comprehension of mathematics concepts by English students schooled in a French immersion high school. Similarly, Lindholm-Leary and Borsato (2003) found that grade 9 through 12 Hispanic students in a two-way bilingual program had more positive attitudes toward and higher performances in mathematics than is normally expected from the average Hispanic students who are considered to be at-risk and those students who come from low socio-economic background. They suggested that this type of program might contribute to this population’s proper academic training and development of positive attitudes required to be more successful in high school. In contrast, Met and Lorenz (1997) reported teachers' concerns about fifth and sixth grade students in partial immersion programs, who are faced with academic challenges as the curriculum content becomes more abstract. These students' proficiency in the language of instruction might not be at an appropriate level for them to engage in academic tasks that require higher level cognitive skills in that language (e.g., reading comprehension, discussion, and writing).

A strategy that has proven successful to address both language and academic development in a linguistically diverse classroom is the use of students' knowledge of both languages. In a study of five classrooms of limited English proficient (LEP) and non English proficient Hispanic populations, Khisty (1995) found patterns of discourse in which teachers emphasized word meanings by varying voice tone and volume or drawing direct attention to a specific word. They also used "recasting" where the teacher correctly restated errors and gave ample opportunity for using the new words in sentences and giving synonyms. One important result in the study was that although all teachers used both languages, the teacher that engaged the most in explanations, questions and cues to foster students' talk of mathematics, was the one who used Spanish the most. Her tendency was to balance the use of both languages. She would not code switch in the middle of sentences (with the result of unfinished sentences), but rather would use either language, in what can be considered discourse episodes in one language or the other. Thus, her discourse pattern was characterized by complete thoughts rather than broken discourse characteristic of constant code switching. This teacher differed from the other teachers in linguistic background in that she was a native speaker of Spanish. The others were not native Spanish speakers, and they expressed a lack of command of the technical language (i.e., mathematics terminology in this case) required to be able to explain the subtleties of words meaning in day to day versus mathematics contexts. Similarly, Langer, Bartolome, Vazquez, and Lucas (1990) found that fifth-grade students coming from bilingual homes (Spanih; English) use their Spanish language competence to support their performance in reading school materials in both languages. Reese, Garnier, Gallimore and Goldenberg (2000) found similar results in a longitudinal study in which English reading acquisition was supported by early literacy development prior to entering kindergarten regardless of the language use at home. These results parallel those found in immersion studies that have proposed fostering the development of interlanguage through pedagogical approaches that may include teaching strategies that use the students' first language (Lyster, 1990). Thus, it is important to consider the sociolinguistic and cognitive needs of students in culturally diverse classrooms, based on what they bring into the classroom from the home context. Furthermore, students themselves will use their knowledge of both languages to overcome any difficulty they may encounter in academic tasks (Cohen, 1994; De Courcy & Burston, 2000). In other words, linguistic and cultural diversity in the classroom should be considered a didactic resource rather than an obstacle.

1.3 Culturally diverse classrooms

There is ample evidence that children learn mathematics in similar ways regardless of their cultural background (Carey, Fennema, Carpenter & Franke, 1995). However, within this framework of universality of mathematics learning one of the main assumptions is that students must be able to connect prior knowledge to new mathematics knowledge. Prior knowledge includes an intuitive knowledge of basic mathematics concepts across cultures that children bring with them when they start school, such as counting and modeling strategies that they develop to make sense of their surroundings. This intuitive knowledge forms the basis for the development of the formal mathematics knowledge they will acquire at school, such as symbols, computational procedures, and abstract concepts (Hiebert & Carpenter, 1992).

The Cognitively Guided Instruction (CGI) program of Fennema, Franke, Carpenter and Carey (1993) is based on the notion that school-based knowledge is built on the knowledge students have prior to coming to school is. Knowledge development within this framework takes into account the child's history of experiences which are based on the context/culture in which the child has been growing up. Teachers who teach mathematics using CGI have students solving problems most of the time, and these problems are usually based on something relevant in the students' life, something students might be studying outside mathematics, or on a book the teacher may have read to them. The authors report that compared to classes who did not use CGI in their teaching of mathematics, children in the CGI show more flexibility, creativity in their choice of solving procedure, and better fluency in reporting their results.

With respect to word problems CGI training allows teachers to get away from the traditional word problems that are presented in textbooks. Textbook problems do not offer much of a relevant context to the children, which might be an obstacle to their creativity and flexibility in their problem solving strategy. In contrast, problems that emerge from within the class, based on the students' experiences or real world events, or literature read and discussed in class, are more motivating. Students will see the problems as more relevant to their daily lives which in turn will make it easier to engage them in problem solving. In a similar view, Roth (1996) argued that word problems do not become contextualized simply because the mathematical information is presented within the framework of a story. The story should be related to mathematical experiences lived by the students in the real world. In general, this is not an opportunity all students have before having to solve word problems. Additionally, he suggests that when faced with word problems, students have to deal with what he refers to as the "commonsense knowledge it takes to understand (any) text" (p. 520), which has to do with the goals of the writer and the assumption of competent reading skills on the part of the reader.

Despite the above-mentioned results and the acknowledgement of the role of students' linguistic abilities in learning academic subjects, there still exists the notion that mathematics teaching and learning transcends linguistic considerations. Khisty (1995) argued that this notion has contributed to a lack of proper attention to issues of classroom discourse and learning environments that foster students talk of mathematics when it comes to Hispanic students' performance in mathematics. Fillmore and Snow (2000) proposed that training programs for teachers should include courses in educational linguistics that emphasize knowledge about language and its use in formal and academic contexts. This should contribute to the teachers' better understanding of the pivotal role that language has and its use in their practice especially in culturally heterogeneous classrooms.

To summarize, teachers in culturally and linguistically diverse classrooms must have as their ultimate goal the students’ development of a good command of the appropriate mathematics discourse that would allow them to have a sense of what it means to do mathematics in any language. Therefore, they should foster the enculturation into the area in such a way that feelings of being alienated from mathematics are discouraged. In this sense, mathematics teaching is viewed as being participatory and socially constructed. Kelly and Bretton (1999) have made similar suggestions in their study in science courses in bilingual classrooms. These views are congruent with the recent proposals for mathematics teaching and learning within a discursive or communicative framework (Kieran, Forman, & Sfard, 2002) in which the inherent social nature of human thought is acknowledged. They view knowledge construction as an activity that develops through students "participation in substantive conversations about meaning and uses of language" (p. 18). Within these perspectives the development of expertise in the discourse of a scientific community is viewed as the constant participation of the learner, where competency is expressed through actions and words in the constant exchanges with members of the community of interest (Kieran et al., 2002; Pea, 1993).

The present study study investigated the role of students' linguistic skills in their L1 and L2 when they were reading and attempting to solve algebra word problems. Detailed analyses of the problems' text characteristics, use of language, the story embedded in the problem, and the mathematics content were performed in order to assess the different levels of understanding that may be required to perform the task. The solution to the problems or outcomes of students' performance, and their responses to a post task interview were also analysed. The students' think-aloud protocols were analysed through two coding schemes with the purpose of seeking the process students go through in comprehending and representing the problems. The results presented in this symposium focused on the results based on the students’ performance on the problems’ final solutions, although a summary of the results on the think aloud protocols is given as they pertain to the role of language.

2. Method

2.1 Setting

The present study was conducted at a private bilingual school that serves families of high socio economic status in a Colombian city. The school prides itself on providing bilingual education where students learn Spanish and English within the Spanish-speaking context and culture as early as kindergarten. Mathematics instruction is exclusively in English from elementary 1 up to grade 9, although the school stresses the use of either language when needed for conceptual comprehension. After grade 9 students are switched to Spanish instruction to start preparing for the national examination required for University entrance in grade 11. Thus, the school's bilingual system falls within the characteristics of a partial immersion program (i.e., the L2 is the main language of instruction although there are a few subjects that are taught in the first language) the curriculum parallels the local L1 curriculum, there is overt support for the use of L1, exposure to the L2 is largely confined to the classroom, students enter with similar levels of proficiency in L2, the classroom culture is that of the local L1 community (Swain & Johnson, 1997).

2.2 Participants

The sample consisted of 31 grade 9[1] Colombian students, 19 boys and 12 girls, who volunteered to participate in the study.

The school divides grade 9 students in two groups based on their mathematics records and L2 standardized tests a core group and an extended group. Academic records and L2 tests indicated that both groups have high L2 proficiency, but differ in mathematics performance. The core group records showed medium to low mathematics performance, whereas the extended group showed high mathematics performance. Students in the core group were receiving mathematics instruction in English from a British teacher. The extended group followed a more advanced curriculum and were already receiving instruction in Spanish. Although this group division is mainly based on subject matter tests, results on L2 tests are taken into account, to ensure that low academic performance is not due to linguistic factors. Of the students participating in this study, there were 15 students in the core group and 16 students in the extended group.

2.3 Materials

Materials consisted of a warm-up problem and four problems that dealt with two topics already covered in the curriculum, namely, *ratio* and *percentage*. Two problems for each topic were chosen from the students' English mathematics textbook (Coxford & Payne, 1987; 1990), in consultation with the teachers. These problems were translated into Spanish by the experimenter to allow for presentation in either language (See Appendix A). The four problems were presented to students on the same sheet of paper (8.5 x 11 in., 16-point font) along with extra paper, pencils and erasers. A warm-up problem was presented in English on a separate sheet of paper.

2.4 Design

The present study used a mixed two-factor design that involved one between-subjects factor, namely, group membership, and two within-subjects factors, namely problem topic and language of presentation. Thus, the independent variables were group, topic, problem within each topic and language of presentation. The dependent variables were the accuracy of problem answers, the responses given by the students in a post task interview session, and the text comprehension measures on the students' think-aloud protocols.

Students were asked to solve four problems, two in English and two in Spanish. Within each language, one problem was a ratio problem and the other was a percentage problem. The order of presentation was counterbalanced with four different sequences based on the initial sample of 40 students. The order of language of presentation was kept constant. That is, the first two problems were always in English and the final two problems in Spanish. Although the participation of each student in each session was random, the presentation of each sequence was given in order as the students from each group came to the session. That is, the first five students from each group received the first sequence, and so on. Since nine students dropped out of the study (four from the extended group and five from the core group), the counterbalance procedure was carried out as illustrated in Table 1, which gave the result presented in Table 2.

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[1] Grade 9 was selected because it is at this level that students start doing word problems that are more complex in terms of the information given in a problem and the number of steps to be taken to solve the problem. A final decision to keep this grade for the study was made based on the curriculum in Colombia.

Table 1
Counterbalance Procedure

Sequences	Order of Problem Presentation 1	2	3	4
I S_1, S_2, S_3, S_4, S_5, S_{17},S_{18}, S_{19}, S_{20}, S_{21}	P1L2	R1L2	P2L1	R2L1
II S_6, S_7, S_8, S_9, S_{10}, S_{22}, S_{23}, S_{24}, S_{25}, S_{26}	R1L2	P1L2	R2L1	P2L1
III S_{11}, S_{12}, S_{13}, S_{14}, S_{15}, S_{27}, S_{28}, S_{29}, S_{30}, S_{31}	P2L2	R2L2	P1L1	R1L1
IV S_{16}	R2L2	P2L2	R1L1	P1L1

Note. S: Subject (S1 to S16: Extended, or high mathematics ability, Group; S17 to S31: Core, or low mathematics ability, Group). P1: Percentage problem 1. P2: Percentage problem 2. R1: Ratio problem 1. R2: Ratio problem 2. L1: Spanish. L2: English

Table 2
Result of Counterbalance Procedure

Sequences	Order of Presentation				Number of observations	Number of Students
	L2 (English)		L1 (Spanish)			
I	P1 *n*=10	R1 *n*=10	P2 *n* =10	R2 *n* =10	40	10
II	R1 *n* =10	P1 *n* =10	R2 *n* =10	P2 *n* =10	40	10
III	P2 *n* =10	R2 *n* =10	P1 *n* =10	R1 *n* =10	40	10
IV	R2 *n* =1	P2 *n* =1	R1 *n* =1	P1 *n* =1	4	1
Number of Observations	31	31	31	31	124	31
	62 observations		62 observations			

The reduction in the number of students meant that the P2 and R2 problems were presented 11 times in English, but 20 times in Spanish. By the same token, P1 and R1 were presented 20 times in English, but 11 times in Spanish (See Table 3).

Table 3
Number of times each problem was presented in either language.

Problem	Number of times presented in each language	
	L1: Spanish	L2: English
P1	11	20
P2	20	11
R1	11	20
R2	20	11

2.5 Procedure

A letter of consent in Spanish was distributed to the parents containing a summary of the study, its purpose and procedure. The instructions were given orally in English by the experimenter after the problem sheet was handed out.

The students also received extra blank paper, a pencil, and an eraser. The 45-minute academic period allotted to carry out the task was the time the school administration allowed the students to be absent from class. Students were allowed to use either language when solving the problems, and the use of a calculator if they asked for one. After they finished the task, students answered five questions in a semi-structured interview. The questions were related to the use of the L1 and L2 in solving the problems, solving this type of problems, the texts, etc. The goals of this interview were to have a sense of what their personal thoughts were about the task, and to obtain additional information on the ways the L1 and the L2 were used in order to perform the task (see Appendix B). This part of the session was carried out in Spanish. The sessions were audiotaped for later analysis.

2.6. Problem analysis

What follows are the different levels of the task a student is assumed to go through when performing the task, namely, the text for each problem in both languages (See Appendix A); the text-base or semantic content for each problem presented in Table 4, Table 5, Table 6, and Table 7, respectively (based on Frederiksen, 1975; 1986); a description of the situation model and the problem model represented in each problem, a graphical convention[2] of the problem model, and the solution to the problems.

Table 4
Text Base for Ratio Problem 1

Segment #	Proposition #		
1		In a mixture of concrete, the ratio of sand to cement is 1:4.	
	1.1	Mixture	CAT: Concrete TENSE: Present.
	1.2	Mixture	PART: Sand (QUANT: 1)
	1.3	Mixture	PART: Cement (QUANT: 4)
	1.4	FUNCTION:RATIO	[sand, cement] RETURNS: [1, 4]
2		How many bags of cement are needed to mix with 100 bags of sand?	
	2.1	Needed	PATIENT: EMPTY OBJECT: Cement UNIT IDENTIFIER: Bag TOKEN NUM: PLURAL DEGREE: Many (INTERROG: How) TENSE: PRESENT GOAL: 2.2
	2.2	Mix (with)	AGENT: EMPTY OBJECT: Sand DEGREE MEASURE REAL: 100 UNIT IDENTIFIER: Bag. TOKEN NUM: PLURAL

Note. Number of words in English version: 27. Number of words in Spanish version: 29. Number of Sentences: 2. Number of propositions: Sentence 1= 4; Sentence 2= 2; Total=6.

Situation model. The situation presented in the above problem is about how to obtain something physical in the real world, namely, concrete. In terms of general knowledge students have to know that mixture means putting things together; that concrete is different from cement and sand; that concrete is obtained by putting together different amounts of sand and cement.

Problem model. This is a part part problem and in order to elicit the right schema the student has to understand that the concept of ratio entails a comparison between quantities. This is specified in the text base outlined in Table 4, in propositions 1.2, 1.3, 1.4. The student must also know the ratio symbol (:) and how it is read. Additionally, the student ought to understand the word *mixture* as an addition mathematical concept. In terms of the problem, the first sentence of the text omits the unit of measure "bag". Thus, in order to grasp the meaning of this sentence the student has to know or understand that in the ratio expression 1:4 the digit on the left hand side of the ratio represents the sand and the one on the right represents the cement, and that those values represent a unit of measure. The students should also know that the ratio can also be expressed as a fraction. Although the problem deals with three objects, there is only one unit of measure (bag) that can contain either one of the two categories of objects that form the third one. Finally, the students have to understand that they have to find an equivalent ratio to the one presented in the text.

The problem model may be represented as in Figure 1 below.

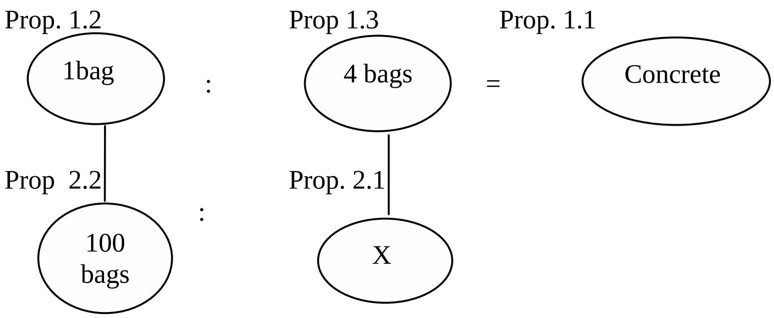

Figure 1. Model based on ratio schema for Ratio Problem 1

Equation and Solution:

$$\begin{array}{lcl} 1 : 4 & \text{or} & \frac{1}{4} = \frac{100}{X} \\ 100 : X & & \\ 100 : 400 & & \frac{1}{4} = \frac{100}{400} \end{array}$$

Table 5

Text Base for Ratio Problem 2

Segment #	Proposition #		
1		Henry and Alice decide to divide a profit of $400 in the ratio of 5:3	
	1.1	Decide	AGENT: Henry, Alice RESULT: 1.2 TENSE: PRESENT
	1.2	Divide	AGENT: (EMPTY) THEME: Profit NUMBER: DEF
	1.3	Profit	QUANT: $400
	1.4	FUNCTION: RATIO	OPERAND: Profit RETURNS: [5:3]

Note. Table continues on the next page.

Table 5 (Continued)
Text Base for Ratio Problem 2

2		If Alice receives the larger amount,	
	2.1	Receives	RECIPIENT: Alice THEME: Amount DEGREE: larger DEF NUM: SING
	2.2	COND: If	[2.1] [3.1]
3		How much does each receive?	
	3.1	Receives	RECIPIENT: Each PART: Much (INTERROG: How)

Note. Number of words in English version: 24. Number of words in Spanish Version: 25. Number of Sentences: 3. Number of propositions: Sentence 1= 4; Sentence 2= 2; Sentence 3: 1;Total = 7.

Situation model. This problem presents a situation between two people who have obtained a profit. This is essentially the only general knowledge concept the students should understand, the meaning of a profit. One difference with the previous ratio problem is that in this one there is one unit measure (i.e., money) that is divided between two people. The two terms in the ratio expression refer to money. Finally, they also have to understand that it is the woman who receives the larger amount, but she is mentioned second in the problem. This can only be inferred after reading segment 2.

Problem model. This is a part-whole problem. In this problem a specific amount, is divided into parts expressed in the ratio. In terms of mathematical understanding most of the reasoning in the previous ratio problem applies. The student has to understand that the concept of ratio entails a comparison between quantities. The basic arithmetic concept of the division of the $400 should be straightforward. The student must know the meaning of the mathematical symbol (:) and how it is read. In terms of the ratio expression 5:3, the student has to understand that the digit on the left hand side of the ratio represents the amount Alice receives and the other one represents the amount Henry receives. The problem deals with three amounts of the same unit of measure, money. Finally the students have to understand that they have to find the respective amounts expressed in the ratio whose addition is equal to the $400. The problem model may be represented as in Figure 2.

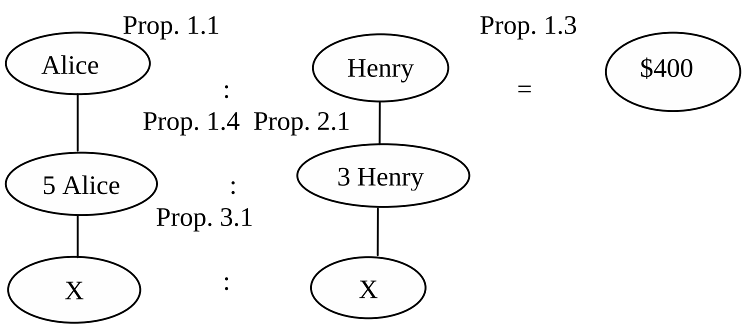

Figure 2. Model based on ratio schema for Ratio Problem 2

Equation and Solution:

$$5X + 3X = 400$$
$$8X = 400$$
$$X = 400/8$$
$$X = 50$$
$$5X = 5(50) = \text{Alice receives } \$250$$
$$3X = 3(50) = \text{Henry receives } \$150$$

Table 6
Text Base for Percentage Problem 1

Segment #	Proposition#		
1		Jeff earns $1050 a month as keypunch operator for a bank	
	1.1	earns	PATIENT: Jeff,
			THEME: Income 'elided'
			TENSE: Present
	1.2	'Income'	QUANT: $1050
			DURATION: Month
			ASPECT: ITERATIVE
	1.3	CONDITION	1.4
	1.4	Work 'elided'	AGENT: Jeff
			CATEGORY: key punch operator
			RECIPIENT: bank
			ASPECT: CONTINUOUS
2		He spends $84 per month on his automobile insurance.	
	2.1	spends	AGENT: He,
			THEME: Money 'elided'
			TENSE: Present
	2.2	'Money'	QUANT: $84
			DURATION: Month
			GOAL: 2.3
			ASPECT: ITERATIVE
	2.3	POSSESS	PATIENT: His
			OBJECT: 2.4
	2.4	Insurance	CAT: Automobile
3		What percent of his monthly income is spent on automobile insurance?	
	3.1	Spent	AGENT: EMPTY 'He'
			THEME: 3.2
			GOAL: 3.3
			TENSE: Present
	3.2	Income:	PATIENT: His
			ATTRIBUTE: Monthly
			PART: Percent (wh question)
	3.3	Insurance	CAT: Automobile

Note. Number of words in English version: 31. Number of words in Spanish Version: 34. Number of Sentences: 3. Number of propositions: Sentence 1= 4; Sentence 2= 4; Sentence 3: 3; Total=11.

Situation model. In the present problem the situation could be described as the following: Jeff works (this slot is empty) for a bank as a key punch operator. Although being a key punch operator is not relevant to the solution, this is not an activity adolescents can relate to. Jeff is earning, getting paid, or receiving a salary of $1050 monthly. He pays $84 every month to cover his automobile insurance. Although students may know the word *insurance*, it is not something they may have had to deal with at this age. The student reader must infer that the insurance is paid from the salary received from the bank, and that the payment to the insurance company is made after receiving that salary. This can only be inferred after reading the problem's question.

Problem model. After reading segments one and two, the reader may elicit a subtraction schema (See proposition 1.1 to proposition 2.4 in Table 6), suggesting the use of the key word schema elicited by the word *spend.* This schema should change right after reading the problem question, or segment three in the problem's text base, which should elicit the percentage schema. Should the student stay with the schema of subtraction and decide to find the percentage through it, the solution would be longer and thus more complex. But should the student change to the right schema (percentage) from the beginning, the problem should be solved faster. Figure 3 presents the problem model based on the percentage schema, whereas Figure 4 presents the problem model based on the percentage and subtraction schemas.

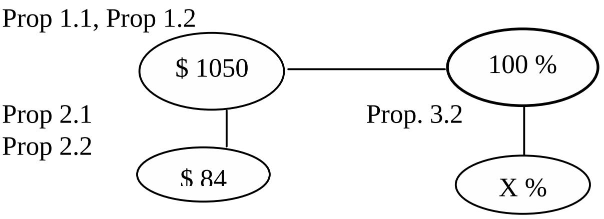

Figure 3. Model based on percentage schema for Percentage problem 1

Equation and solution:

$ 1050 = 100%
$ 84 = X
X= 84 / 1050 x 100% = 8%

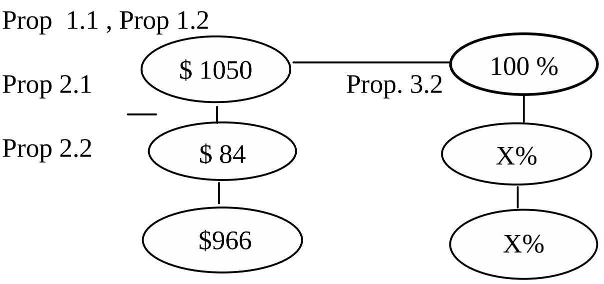

Figure 4. Model based on subtraction and percentage schema for Percentage problem 1

Equation and solution with subtraction schema:

X=1050 - 84 = 966
966 x 100% / 1050 = 92%
X= 100% - 92 % = 8%

Table 7
Text Base for Percentage Problem 2

Segment #	Proposition#		
1		Claire has twice as much money invested at 9% as at 12%.	
	1.1	POSS	PATIENT: Claire OBJECT: Money TENSE: PRESENT.
	1.2	Money	PART: [1.3], [1.4]
	1.3	Money	ATT: invested (ATT: rate ‘elided’) DEGREE MEASURE REAL: 9% TENSE: PRESENT
	1.4	Money	ATT: invested (ATT: rate ‘elided’) DEGREE MEASURE REAL: 12% TENSE: PRESENT

Note. Table continues on the next page.

Table 7 (Continued)
Text Base for Percentage Problem 2

	1.5	FUNCTION: MULT	[1.4] DEGREE MEASURE REAL: 2 RETURNS: 1.3
2		Her annual interest on the money at 9% is $1140 more than her annual interest on the money at 12%.	
	2.1	FUNCTION: DIFF (More than)	[2.3], [2.5] RETURNS: $1140
	2.2	Interest	REC: Her SOURCE: Money (1) DURATION: Annual TENSE: PRESENT.
	2.3	Money (1)	MEASURE: Rate ‘elided’ DEGREE MEASURE REAL: 9 UNIT IDENTIFIER: PERCENTAGE (%) RESULT: 2.2
	2.4	Interest	REC: Her SOURCE: Money (2) DURATION: Annual TENSE: PRESENT.
	2.5	Money (2)	MEASURE: Rate ‘elided’ DEGREE MEASURE REAL: 12 UNIT IDENTIFIER: PERCENTAGE (%) RESULT: 2.4
3		How much is invested at each rate?	
	3.1	Invested	AGENT: 'EMPTY' OBJECT: WH QUESTION MEASURE: Rate TOKEN NUM: Each. TENSE: PRESENT

Note. Number of words in English version: 39. Number of words in Spanish Version: 40. Number of Sentences: 3. Number of propositions: Sentence 1= 5; Sentence 2= 5; Sentence 3: 1; Total=11.

Situation model. The situation presented in this problem could be described as follows: the student has to understand that Claire has two different amounts invested in a financial institution. In one of the accounts, which she has at an interest rate of 9%, she has double the money than in the other account, which is at an interest rate of 12%. Thus, students have to understand the meaning of interest on investment. Also, the student has to understand that after a year Claire receives $1140 more in interest on the money at 9%, than on the money she has invested at 12%. Therefore they have to know that this is the difference in interest between the two amounts. At this point the student should be able to conclude that the money at 9% is the larger one. The student has to find the amount invested at each rate. Although this is not a situation that the average adolescent would encounter at their age, they might be able to relate to it.

Problem model. This is a multi-step problem that requires two equations to solve it. To construct or elicit the right schema, the students have to know or understand the concept of investment, the concept of interest rate on investment and how they relate it to the concept of percentage. Additionally they have to understand that the expression *twice as much as* means that the money at 9% is double the money invested at 12%. They also have to understand that the $1140 is how much more she earns on the account at 9% when compared to the other account's earning at the end of year, and not what she actually gains in that account. The expression *more than* in this segment should elicit the difference schema (i.e., proposition 2.1 in Table 7). Finally the students need to understand that they have to find the two amounts, and that by solving for one they will find the other. This should be solved by doubling one account and adding \$1140. Figure 5 present the problem model.

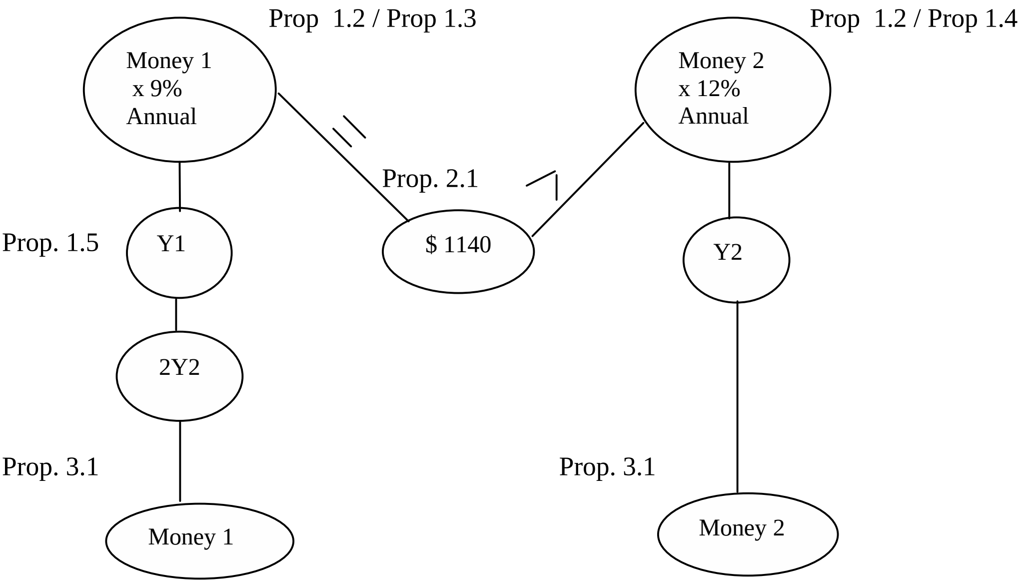

Figure 5. Model based on percentage schema for Percentage problem 2

Equation and Solution:

$Y_1 = M_1 \times 9\%$ $\quad$ $Y_2 = M_2 \times 12\%$ $\quad$ $M_1 = 2M_2$
$Y_1 = Y_2 + 1140$
Substituing for Y's:
$M_1 \times 9\% = (M_2 \times 12\%) + 1140$
$2M_2 \times 9\% = (M_2 \times 12\%) + 1140$
$18\% M_2 = 12\% M_2 + 1140$
$6\% M_2 = 1140$
$M_2 = 1140/6\%$
$M_2 = \$19000$
$M_1 = 2M_2 = \$38000$

Finally, after the sessions started the experimenter realized that this problem missed one piece of information in the Spanish translation, which is key to its solution. Although the translations were checked by two other individuals at the school, the error was nonetheless missed. Notice the error below:

Claire has **twice as much** money invested at 9% as at 12%. Her annual interest on the money at 9% is \$1140 more than her annual interest on the money at 12%. How much is invested at each rate?

Clara ***tiene (el doble de) dinero*** *invertido tanto al 9% como al 12%. Su interès anual en el dinero al 9% es de \$1140 màs que su interès anual en el dinero al 12%. Cuànto ha invertido en cada una de las ratas?*

The text in bold in the English version indicates the words that should have been translated in the Spanish text. The bolded Spanish text in parenthesis indicates where the translation of these words should have been inserted. Despite this unfortunate mistake, the problem was kept in the study as it was. Not only had the sessions already started when it was found, but the mistake was not considered an obstacle for the students to demonstrate a measure of their text comprehension. Additionally, it was decided that it was important to investigate whether these students would still attempt to solve the problem, as other students have tried in other studies with arithmetic word problems in which they have received hints that the problems might not have a solution. Thus, the students may attempt solutions that may not be situational, mathematical or logically realistic (Greer, 1997; Reusser & Stebler, 1997; Yoshida, Verschaffel, & de Corte, 1997; Wyndham & Saljo, 1997). Furthermore, Rehder (1999) found similar results with unsolvable algebra word problems. However, in his study, hints as to whether a problem was unsolvable or not, served as a good strategy to improve detection of unsolvable problems for students with moderate mathematics ability, when the story was familiar. In contrast, a solvable problem was more likely to be classified as unsolvable when the story was unfamiliar. High ability students correctly detected solvable and unsolvable problems, regardless of the story.

3. Results

The focus of this study was to investigate the relationship between language and mathematical ability by looking at the case of grade 9 students who have received their mathematics instruction in their L2, within a context in which their L1 is the language of the majority. The study was designed to address the more specific issues of whether current explanations in the literature for the difficulty of word problems in a student's first language would apply to students who have been learning mathematics in their L2, how these explanations would apply to the bilingual case, and if there were any other factors that would affect the difficulty of word problems for the bilingual student.

Therefore, the results presented in this paper are based on analyses carried out to answer the following questions:

Does language of presentation have an effect on arriving at the problem solution (i.e., incorrect or correct answers to problems)?

Does problem topic (i.e., percentage and ratio) have an effect on arriving at the correct solution to the problems?

Is there a difference in solving the problems regardless of topic?; that is, was solving the problem a function of individual problems?

Does language of problem presentation within a topic affect problem solution?

Do students have a preference for either language when performing this type of task?

Does belonging to the extended or core group have an effect on task performance?

It was hypothesized that there would be a significant effect of language of presentation when reading and solving the problems. That is, understanding the text in order to put it into the language of mathematics and solve it should be significantly harder when doing it in the L2; and solving the problems would be affected by the way the problem was comprehended.

The results presented in this symposium are focused on the analysis of the accuracy of solutions to problems and the results from the post task interviews. However, the students' think-aloud protocols were also analyzed through two different coding schemes, namely, a descriptive coding scheme and an inferential coding scheme. A summary of the results from those analyses will be given as they pertain to the results presented here.

3.1 Accuracy of solutions to problems

The goal of this analysis was to investigate the pattern of final solutions to each problem with respect to language of presentation. Thus, the analysis was performed by problem. As stated in the Method section the second percentage problem (i.e., P2) was mistranslated. This put the students who received it in Spanish at a disadvantage. However, none of the students who received it in English came up with the correct solution to the problem either. Since this was an analysis of the students' final solution to each problem, it was decided not to include it at all for this part of the analyses.

The total frequency count of correct or incorrect final answers to the three problems included in this section were used for the statistical analyses. Overall, there were more incorrect (56%) than correct solutions to problems (44%). Figure 6 presents the percentage of those solutions in each language. More correct solutions were given when problems were presented in the L2 (English), whereas more of the incorrect solutions occurred when problems were presented in the L1 (Spanish), showing a language effect.

A 2 x 3 log linear analysis was performed to further investigate this language effect and the solutions to the three problems. The analysis revealed a significant interaction between language of presentation and problems ($\chi^2_{(2)}$= 7.14, p<.05) and between the accuracy of solutions and problems (2 (2) = 17.03, p<.05).

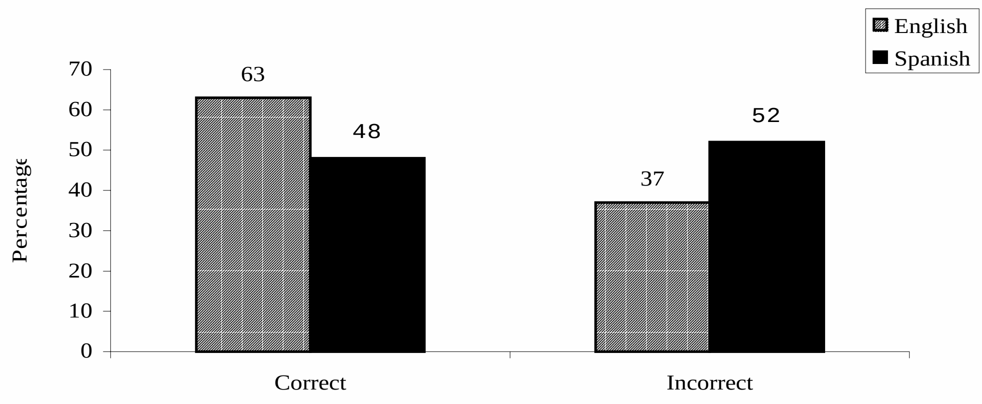

Figure 6. Relationship between Language and Solutions to Problems

Table 9 presents a breakdown of solutions by problem in each language. As can be seen, the first ratio problem (R1) had the highest percentage of correct responses in both languages, although students appeared to have an easier time with the Spanish presentation. In contrast, the percentage problem (P1) and the second ratio problem (R2) showed higher percentages of incorrect solutions in both languages, although the results suggest more difficulty with the Spanish presentation. Thus, when the results are looked at across problems in each language, they suggest more variability when they are presented in Spanish than when presented in English. In summary, these results do not support the hypothesis that solving the problems should be harder in the second language. They suggest that for these bilingual students, in addition to specific features of each problem, the language of instruction may be playing a more important role when performing this type of task. Additionally, the results illustrate a different profile of final solutions across problems, where the pattern of correct and incorrect solutions was different across problems and independent of topic (i.e., ratio or percentage).

Table 9

Percentage of correct and incorrect solutions by language and problem. N=93

Language	Solution	R1	R2	P1
Spanish (L1)	Correct	91%	20%	9%
	Incorrect	9%	80%	91%
English (L2)	Correct	70%	27%	45%
	Incorrect	30%	73%	55%

Finally, a second log linear analysis was performed to investigate the relationship of solutions to problems to group membership (i.e., core vs. extended), which yielded a non-significant effect for group ($\chi^2_{(1)}$= .26, p=.61) and a non-significant interaction with problem ($\chi^2_{(2)}$= 1.69, p=.43), suggesting that arriving at the correct or incorrect answer to the problems was not associated with whether a student belongs to the low or high mathematics ability group.

3.2 Analysis of post task interview data

The answers to the questions in the post task interview were submitted to a frequency count based on the student's answers, and the percentages were calculated. When students were asked their preferences in receiving mathematics instruction in either language, 61% of the students preferred English as the language of instruction, 19% preferred Spanish and the rest did not have a preference. In general, this preference resulted from students' experience with English since kindergarten. This is consistent with previous results that do not support the hypothesis that word problems should be harder in the L2. When asked whether they felt any differences when doing word problems in either language, 45% felt the problems were easier in English compared to Spanish, 22% felt they were easier in Spanish, and 32% did not feel any difference. In this case, the main reason expressed by the majority was vocabulary, especially for key terms. When asked whether they had a preference for word problems or equations, 74% of the students preferred the latter, whereas 16% preferred the former and 9% did not have a preference. The main reason for preferring equations was that "everything is given", whereas in word problems they have to figure out the equation and the computation procedure from the text before attempting to solve the problem. When asked whether they had any difficulty in reading and understanding texts in either language, 32% expressed that they had greater difficulties with texts in Spanish, whereas 26% expressed that they had more difficulty with texts in English. Thirty-two percent said they did not have any difficulty with texts in either language, and 10% said they had difficulty with texts in both languages. Again, these results also suggest that the students feel more comfortable performing tasks in the language of instruction. However, they feel less comfortable with the verbal representation of mathematical problems.

To summarize this section the results showed that the students had less difficulty in solving the problems when they were presented in English, although they made use of both languages in performing the task. These results are supported by the analyses (not presented in this symposium) carried out on the think aloud protocols which showed a high frequency of code switching when problems were presented in English. With respect to the problems themselves, the students also had more difficulty in solving the percentage problems than the ratio problems. Finally, the preferences expressed by the students in the post task interview are consistent with the results. That is, in general they found word problems to be a difficult task, and most of them prefer to do it in English. These overall results are further discussed in the next section.

4. Discussion

4.1 Factors affecting accuracy in problem solutions

The analyses of the students' final solutions to the problems were aimed at answering the questions of whether problem outcomes were influenced by the language of presentation, by the type of problem (ratio or percentage), or by the particular features and content of each problem. Although this part of the analyses is related directly to mathematics performance, it sheds light on the role of the language of presentation as it relates to comprehending the text. A correct problem solution assumes a correct match between the text, the situation presented and the mathematics representation.

The results showed that students arrived at the correct solution more frequently when the problems were presented in English. The results also showed an interaction between the solutions and the problems, thus suggesting that the difficulty students had may also be related to features of the problems themselves. The analytic procedures carried out for each problem and presented in the Method chapter, can help in exploring how the particular text, language use, and mathematics content characteristics of each problem may explain these results.

I present this section below, by first discussing and comparing the results of the three problems that were included in the statistical analyses, namely the two ratio problems (R1 and R2) and the first percentage problem (P1). I end the section with a discussion of the second percentage problem (P2). As the reader may recall, this problem was mistranslated from English to Spanish and the mistranslated piece of information was key to the problem solution. The problem was kept in the study because the students still tried to solve it, and thus, they gave a measure of their comprehension. It could be argued that the problem should not be included at all because the students could not possibly construct a correct problem representation, including the situation presented in the problem. However, the argument for keeping it is related to the aim of the study to investigate the level of text comprehension and representation students attempt in order to solve a problem. Students will try to solve problems without taking into account the realistic considerations. In addition, the students who received the English version of the P2 problem were also not able to arrive at the correct solution. The question arises, therefore, as to the characteristics that may make it more difficult compared to the other problems.

4.1.1 Ratio problems

The R1 problem showed the highest percentage of correct responses in both languages, although students found it easier in Spanish. In terms of propositional content (See Table 4), the text base of this problem has fewer propositions, compared to the R2 problem (See Table 5). In addition, the problem model in R1 also has fewer levels than R2 (See figures 1 and 2). It could be argued that both features, reduced cognitive load in reading and understanding R1. This comparison between the two ratio problems is interesting, because it suggests that the mathematics content and the text semantic structure of the problem had more impact on the students' performance than the situations presented in the problems. It could be expected that the ninth grade students who participated in this study would be more familiar with handling and dealing with money and dividing it, than with making concrete. This is consistent with Sebrecht et al. (1996) who found that problems related to money as the unit of measure were associated with decreased difficulty compared to time conversion and metric measures. In addition, although both ratio problems deal with one unit of measure, 'bag' in the case of R1 and 'money' in R2, R1 deals with three different physical objects, namely, concrete, sand and cement, which may arguably add to the processing load. Finally, in terms of text structure, in R1 the student learns that the ratio numbers refer to bags of each object only when the second segment of the text is read, whereas in R2 the unit of measure is found on the first segment read. Notwithstanding these differences in the situation embedded in the problem and the way it was presented, students did better with the less familiar situation. This suggests that only in the absence of a good mathematical understanding will students turn to the text and situation presented in the problem. Otherwise, it will become less relevant in solving the problem. This suggestion is congruent with what is expected of sophisticated problem solvers who know that the function of the text in word problems is to lead them into the mathematical representation. Nevertheless, as discussed in another section below, students in the present study still relied on making sense of the situation before attempting to build a representation and solve the problems.

An additional point related to the real world situation presented in the R2 problem, is that some students, at the moment of giving the answer, would give the larger amount of money to the male character of the story, when it is the female character, as stated in the text, who receives the larger amount. Although this may be anecdotal evidence, it points to the situation in which inferences are made based on socio-cultural factors, or on the way the information is presented that could lead to content ambiguity. The male character is presented first, but the female character receives the larger amount. One could argue that this representation of the problem is situationally inferred. Thus, although the student ends up with the correct numerical answer, he/she might give an incorrect answer in terms of the situation. This would probably not be a major mistake and did not happen with a high frequency in the present study, but still points to the nature of text comprehension and the impact in a task where the goal is mathematical solution, and not necessarily the recall of a story.

As a final note on the ratio problems some examples that show how the key word strategy may backfire for L2 student attempting to arrive at a problem solution are shown below. The examples are taken from students who were unable to solve the ratio problems. The students confused the word *ratio* with the geometry concept of radius which is also translated as *radio* in Spanish. The following are the specific excerpts from the students' protocol:

Student: I think I have to take the measurement of the ratio...
and... I think that the ratio is pi... ah no.

Yes, pi is 3.14...
aha. So, I... I think I have...
I can’t solve it.
Experimenter: Why?
Student: I’m confusing the circumference and...
the ratio... because I think is 3.14...
and I’m not sure if that...
if that measure is to all the... other formula.

Student: I know what is a ratio.
The half, like the half, for example
[he draws a circle with a radius]
that's a ratio. The circumference.
So the ratio of sand to cement is one four…
But… it's two… the ratio of sand to cement…
It's not too clear,
I don't understand the question.

Student: I know the ratio is… in the… circle,
This diameter [as he draws] this is the…
I don't remember how is called.
The ratio is this… from here to here [pointing to the radius in the circle]
But… I don't know if this has to do with this one [referring to the problem].
Anything else, I understand this,
Alice receives the larger amount [from text]
I have to find how much from the 400 does Henry and Alice receive, but…

This type of error goes back to the situation where a student may be relying on a totally decontextualized key word strategy in order to compensate for the lack of text comprehension and mathematical understanding. In other words, as in many situations where students use some form of trial and error strategy to solve a problem, the student only concentrates on the words, and not the situational or mathematical content presented in the problems. Verschaffel, De Corte, & Vierstraete (1999) offered similar explanations for the failure of fifth and sixth graders to solve nonroutine additive problems. They suggest that this failure may be due to a superficial approach to these problems, that does not allow the students to realize that a routine based solution cannot apply. However, the approach might be taken to compensate for overall lack of understanding.

4.1.2. Percentage problems
4.1.2.1 Percentage problem 1 (P1).

The P1 and the R2 problems appeared to be more difficult than was the R1 problem for the students in both languages of presentation. However, there was more variability in the Spanish presentation. That is, the pattern of performance on the problem solutions in the English presentation tended to be more similar across problems in this language. This pattern of performance adds support to the interaction between problem and language of presentation. As far as the semantic content is concerned, the P1 problem should have been the most difficult problem in both languages of the three included in the analysis (See Table 6) with 11 propositions. In addition, although the mathematics model should not have represented much difficulty if approached directly, the problem solution could also be based on a subtraction schema. This approach leads to more steps for the solution or a completely incorrect solution. The fact that the problem states that Jeff takes an amount from his salary to spend on the automobile insurance, made some students think right away about subtraction; however, the problem is asking for the percentage represented by the $84. Those students who chose the subtraction schema did not go beyond the subtraction. That is, they did not solve for the percentage of the result of the subtraction and thus could not arrive at the correct answer. By choosing the wrong schema they could not solve the problem, although they could have combined the subtraction schema with the percentage schema to solve the problem. In addition, by using the subtraction schema they added an extra level of calculation.

One feature that the P1 and R1 problems in this study share is the fact that the situation in both of them is associated with events or objects that do not belong to the day to day activities or issues adolescents have to confront, e.g., being a key punch operator or making concrete out of sand and cement. Thus, the context information can act as distracters, even though comprehending the situation is not necessary for the actual mathematical solution to the problems. Although a student does not have to know the meaning of every single word to come up with a solution to a problem, the words that are not familiar to the students represent unneeded distracters, as do unfamiliar situations or activities presented in a problem. Fennema, et. al. (1993) reported the ways in which teachers trained in CGI used children's experiences that were shared in the classroom as an important means to develop meaningful mathematics. In their examples, word problems and other problem solving activities were contextualized within stories told in the classrooms. In terms of the situation presented in the P1 problem, the student has to make more inferences than in either ratio problem (See analysis of the situation model of both problems in the analytic procedures in the Method section). Nevertheless, the P1 problem proved to be less difficult in the English presentation than the R2 problem.

4.1.2.2 Percentage problem 2 (P2).

There are a few features of this problem that are worth discussing, notwithstanding its mistranslation and, in consequence, its non-inclusion in the statistical analysis. The expression *twice as much as* proved to be confusing for some of the students. It is a grammatically confusing construction and as such, it represented a distracter to the students. Clement (1982) found similar results with a similar expression, i.e., *times as many*, where students showed reversal errors given that the preposition *as* could be misinterpreted as meaning "equal". Some students in the present study would realize that one account had more money than the other one, only after they read the expression *more than* in the second sentence.

The expression *twice as* was mistranslated in the Spanish version of the problem. The proper Spanish translation of the sentence *Claire has twice as much money at 9% as at 12%*, which proved to be confusing in English, is: *Claire tiene el doble de dinero invertido tanto al 9% como al 12%*, which is also ambiguous. This is an example where the problem difficulty could have been reduced by using a less ambiguous expression. It is also an example where the use of language in word problems, makes the task a test of deciphering language ambiguity, rather than an assessment of mathematical knowledge. This argument about the difficulty of this expression may be seen as support for classroom discourse perspectives that propose that teachers should emphasize having students learn and practice words and expressions within the context of mathematics.

In terms of the semantic content, the P2 problem had 11 propositions, as did the P1 problem. But the situation, the mathematical representation, and solution required more cognitive effort on the part of the student, compared to the P1 problem. On the other hand, the P1 problem contains more semantic information in the last sentence (i.e., the problem question), compared to the P2 problem, which has most of the information concentrated in the first two sentences or problem statement. In the P2 problem the student may be able to infer the question before reading it in its entirety, whereas that is not possible in the P1 problem. With respect to the situation, although the students had already studied interest and percentage in their classes, they did not appear to have a good grasp of the meaning of interest on investment, and how that translates into mathematical terms. Because of the way the situation is presented in the problem text and the ambiguity of the expression *twice as… at… as at*, most of the students realized that the account at 9% was the one with more money only after they read the second sentence that dealt with the amount received in interest. Although, this was the part that was mistranslated in Spanish, it is also ambiguous in that language. Thus, the way the information is presented and the ambiguity of the way language is used added to the complexity of this problem. In a similar view on the role of numeral classifiers in solving arithmetic word problems Miura (2001) discusses how linguistics cues from languages make a difference in building a cognitive representation of a problem. In comparing Japanese and English students, she argued that Japanese students do not have to confront language ambiguity given by the omission of referents in a word problem since Asian languages do not allow this type of ambiguity. In contrast, the English reader have to learn how omitted referents are indicated in a problem.

With respect to its mathematical representation and solution, the P2 problem is also a more complex problem. It requires the student to come up with two equations in order to be able to solve it, whereas P1 requires only one. As can be seen in Figure 5 in the Method section, the model for this problem involves different levels for representing it. This problem is an example in which the complexity of the mathematics content and representation is compounded by the complexity of the text and language use to present it. None of the students in the study who received the English version were able to solve it.

It is important to point out that although the above discussion of the results on the students' solution to the problems suggest that mathematics ability was not the main factor in arriving at the correct solution (i.e., belonging to either group did not have an impact on arriving at the correct problem solution), one cannot disregard the effect of the level of domain relevant knowledge and understanding students must have acquired in order to understand and perform academic tasks. Chen and Donin (1995) found that both L2 proficiency and domain relevant knowledge had an effect on reading time for L2 engineering and biology students. With respect to word problems sophisticated problem solvers would not be confused or distracted by the density or ambiguity of the language use in word problems. That is, they know the story or the situation presented in the problem or this form of representation is used as an artifact for them to express their mathematics knowledge in a more "real" or "authentic" situation than what is reflected in an equation. The results from present study did not find support for this claim. It was previously argued in this section in the discussion of the ratio problems that the results suggested that in the absence of a good mathematical understanding will students turn to the text and the situation presented in the problem. Otherwise, the text and its situational content will become less relevant in solving the problem.

4.2 Interview data

The post task interview data were collected with the purpose of obtaining students' views and preferences related to the task and to mathematics learning in their first and second languages. The group of students who participated in the study were accustomed to mathematics instruction in English. That is the language of instruction they preferred. They also expressed preference for performing the task in English over Spanish. The findings of the study and the responses to the interview questions suggest that it is the language of instruction that is playing a role rather than a facilitating effect of performing the task in one's first language. Moreover, these results are important within the bilingual framework given that the context surrounding the students is that of the first language. The students' school prides itself of offering a bilingual education where students' knowledge of both languages is used in the classroom. The ultimate goal is conceptual understanding; thus, the use of the first language is supported and fostered by the school's administration, so that academic learning is never compromised. This school's policy supports the view that classroom discourse and how language is used in it, is instrumental for students learning, especially in subjects like mathematics where grasping and understanding the subject matter requires high levels of abstraction and reasoning. As discussed in introduction, the use of students’ knowledge of both languages in the classroom has been successful for both language and academic development (Cohen 1994; De Courcy & Burston, 2000; Khisty, 1995; Langer et al., 1990; Reese et al., 2000).

Finally, the students in this study expressed their preference for equations over word problems. Friedlander and Tabach (2001) pointed out that, in general, most mathematics teachers recommend the use of multiple representations in teaching and learning algebra for a better and deeper understanding. In addition, fostering the use of multiple representations is dependent on the presentation of the problem situation and the nature of the questions asked. In their study of a multiple representation of a problem about money saving, presented to seventh graders in normal classroom activities, they found that the students' responses supported their claim that appropriate problem presentation and questioning about it leads to greater awareness of the different ways a problem can be represented in its solution. As in the present study, students preferred the numerical representation, which is normal in beginning algebra. They concluded that students' choice of representation is based on a number of factors and their combination, such as students' style of thinking, simple personal preference to overcome difficulties with other types of representation, and the nature of the problem. They also pointed out that the goal of using verbal representation is to allow the students to see the link between mathematics and other academic areas, as well as everyday life situations. However the ambiguity of language, its lack of universality as compared to mathematical representation, and its reliance on an individual's style may lead to wrong or unimportant associations, as well as hinder the development of the ability to communicate and express mathematics knowledge.

5. Implications and conclusions

The results presented in this study supported the traditional difficulty of word problems regardless of the language of presentation, evidenced on the high number of incorrect solution to the problems. Nevertheless, the majority of the correct solution to the problems were in the students' language of instruction. This result was contrary to what was hypothesized in the study that performing the task in one's first language should facilitate performance in that task. Notwithstanding this result, the students made use of both languages. It is important to point out that these results point to the students' tendency to approach the task with less difficulty in the language they are accustomed to, even if they use both languages in performing it. This is not to say that they are not able to perform it in their first and more dominant language. The relevance of these results is that students will use the resources they have at hand to reduce cognitive load. Thus, the different explanations that exist for solving word problems in the one's first language apply to the students performing in the second language, although the knowledge of both languages and the way it is used in performing the task has to be taken into account. That is, the nature of contextualized L2 learning within the school setting must be taken into account and capitalized upon.

Therefore, the language of instruction should attend to the ways in which students' linguistic abilities are used to their advantage within classroom discourse. These abilities are an important factor in mathematics learning and should facilitate students to have a better grasp of the abstract and complex mathematics language, symbolic system, and computational procedures. In the case of a linguistically diverse classroom, the language of instruction should be used in such a way that it capitalizes on the students' knowledge of both languages. By its very nature, the language of instruction in bilingual settings can vary from subject to subject, activity to activity, etc. With respect to mathematics, the language of instruction should teach students how to speak, comprehend and handle the universality of the language of mathematics.

Khisty (1995) argued for the careful use of language in modeling and scaffolding when teaching new concepts, including mathematics. Using students' L1 in instruction, especially when learning new concepts and to clarify confusions, could be a very effective strategy for language minority students. Given the specificity and meanings of the mathematics terminology, natural language's use depends on the mathematics function. Moreover, mathematics expressions stated in symbols may be expressed in more than one way in natural language; that is, there is a lack of one to one correspondence between the two languages. What is important is to teach the student how to identify mathematics with its specific language. Students should be made aware of the specifics of meaning and uses of language within mathematics contexts.

It is within the context of instruction that students learn and appropriate the language of the area. In pursuing the goal of conceptual understanding and development, the teacher allows the student to become competent, or appropriate the meanings in the discourse of the area taught (Lajoie, 1995; Pea, 1993; van Oers, 2000). The teacher should be aware of letting students know about the particular ways in that day to day language is used within mathematics. This might be problematic for non native speakers of the language of instruction in that they might not be familiar with subtleties in the meaning of words. Examples in the present study came from the confusion some students had with the word ratio and its translation into Spanish, and the meaning of the expression *twice as*.

To end this section, an excerpt from the Principles and Standards for School Mathematics is presented that points to the relevance of effective communication in Mathematics:

> Secondary school students need to develop increased abilities in justifying claims, proving conjectures, and using symbols in reasoning. They can be expected to learn to provide carefully reasoned arguments in support of their claims. They can practice making and interpreting oral and written claims so that they can communicate effectively while working with others and can convey the results of their work with clarity and power (page 1, chapter 7: Standards for Grades 9-12).

Acknowledgement of the role of appropriate general and specific language and discursive skills to develop those abilities is a must, especially for the non native speaker of the language of instruction.

Finally, it is important to point out that these results should be viewed within the framework of the bilingual school setting these students have; it is similar to most bilingual school settings one may find in South America where most countries have Spanish as the language of the majority and English as the official second language. However, from cognitive and discourse perspectives, the findings related to the students' preference for, and less difficulty with the language of instruction, the use of both languages in performing the task are relevant for any bilingual student performing this type of task.

Appendix A

Ratio Problems

R1E[3]: In a mixture of concrete, the ratio of sand to cement is 1:4. How many bags of cement are needed to mix with 100 bags of sand?

R1S: En una mezcla de concreto, la relación entre la cantidad de arena y cemento es de 1:4. Cuántas bolsas de cemento se necesitan para mezclar 100 bolsas de arena?

R2E: Henry and Alice decide to divide a profit of $400 in the ratio 5:3. If Alice receives the larger amount, how much does each receives?

R2S: Enrique y Alicia deciden dividir una ganancia de $400 a razón de 5:3. Si Alicia recibe la cantidad más grande, cuánto recibe cada uno?

Percentage Problems

P1E: Jeff earns $1050 a month as a key punch operator for a bank. He spends $84 per month on his automobile insurance. What percent of his monthly income is spent on automobile insurance?

P1S: Jaime gana $1050 al mes como digitador en un banco. El gasta $84 al mes en el seguro de su carro. Qué porcentaje de su salario mensual se gasta en el seguro del carro?

P2E: Claire has twice as much money invested at 9% as at 12%. Her annual interest on the money at 9% is $1140 more than her annual interest on the money at 12%. How much is invested at each rate?

P2S: Clara tiene el doble de dinero invertido tanto, al 9% como al 12%. Su interés anual en el dinero al 9% es de $1140 más que su interés anual en el dinero al 12%. Cuánto ha invertido en cada una de las ratas?

Warm-up Problem

Two campers leave the same camp and jog on opposite directions. If they jog at 2m/s and 3m/s respectively, how long will it take before they are 3 km apart?

Appendix B

Post task interview questions

1. Cuándo ingresaste a este colegio?
   When did you come to this school?
2. Tienes alguna preferencia en recibir tus clases de Matemáticas en alguno de los dos idiomas?
   Do you have any preference in having your Math classes in either language?
3. Hay alguna diferencia cuando tu haces estos problemas en Inglés o en Español?
   Is there any difference when you do these problems in English or Spanish?
4. Tienes alguna preferencia entre hacer ecuaciones y hacer problemas de enunciado?

[3] R: Ratio. P: Percentage. E: English S: Spanish.

Do you have any preference in doing equations and word problems?
5. Sientes alguna dificultad en leer y comprender los textos en alguno de los dos idiomas?
Do you have any difficulty in reading and understanding texts in either language?

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