The Silent “Number” in Complex Cardinals

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1. Introduction

As shown in (1), a complex cardinal is composed of a multiplier and a multiplicand in many languages (Hurford (1975), Greenberg (1978)).

(1) a. three hundred [English]   b. san byaku [Japanese]   c. tres cientos [Spanish]
    3 100   3 100   3 100
    ‘300’   ‘300’   ‘300’

Ionin & Matushansky (2006) argue that multipliers and multiplicands have the same syntax and semantic. This paper argues that multipliers and multiplicands are different both in syntax and in semantics, contra Ionin and Matushansky’s (2006) unified analysis. More specifically, I propose that a multiplicand is the head of UnitP and can take the silent NUMBER as its complement (Kayne (2005), Zweig (2006)).

2. Ionin & Matushansky (2006): A unified analysis

Ionin & Matushansky (2006) argue that in languages where nouns appear with number morphology, complex cardinals like “300” have the structure in (2).

(2) NP <<e, t>, <e, t>>
  |                 |
N <<e, t>, <e, t>> NP <<e, t>>
  |                 |
three hundred

As for the semantics of cardinals, they propose that a multiplier and a multiplicand have the same semantic denotation, as shown in (3). Π stands for a partition function that takes a cover S and a plural individual x as its argument. S is a cover of x if and only if x is the sum of all members of S. Moreover, Π forbids the relevant elements to be counted twice. The only difference between “3” and “100” is the cardinality of a cover.

(3) a. [[3]] = λP ∈ D_{<e,t>} . λx ∈ D_e . ∃S ∈ D_{<e,t>} . [Π(S)(x) ∧ |S| = 3 ∧ ∀s ∈ S . [P(s)]]
b. [[100]] = λP ∈ D_{<e,t>} . λx ∈ D_e . ∃S′ ∈ D_{<e,t>} . [Π(S′)(x) ∧ |S′| = 100 ∧ ∀s′ ∈ S′ . [P(s′)]]

Of importance here is that multipliers and multiplicands have the same status in both syntax and semantics under the unified analysis. In the next section, I will provide data that are unexpected under the unified analysis.

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3. Problems for the unified analysis

3.1. Left-branch extraction

The unified analysis predicts that a complex cardinal should not behave as a single constituent because a multiplier and a multiplicand do not form a constituent, as shown in (2). However, this prediction is not borne out. In Bosnian/Croatian/Serbian (BCS), “300” can undergo left-branch extraction (LBE), as shown in (4).

(4) \[Tri stotine]_1 je Ivan pozvao [\(\Delta_1\) studenata].
\[BCS\]
3 100.ACC.FEM is Ivan invited students.GEN.MASC
Lit. ‘[Three hundred], Ivan invited [\(\Delta_1\) students].’

Given that LBE can be applied only to a phrasal constituent, (4) shows that a multiplier and a multiplicand form a phrasal constituent, excluding the head noun. Note that je in (4) is a 2nd position clitic; it can follow only one constituent (Bošković (2001), Samuels (2014)). Therefore, (4) cannot be derived by multiple LBE. It is not clear how to deal with this data under the unified analysis.

3.2. Wh-questions

In many languages, multipliers can be questioned by using wh-items. In (5), the topmost multiplier is replaced by a wh-word while keeping the other digits.

(5) **Context:** One day, John told Mary that he invited 8976 students to their wedding party. Two weeks later, Mary forgot the exact number of the invited students, but she can remember digits up to the hundreds place.

a. Maria zastanawia się [iliu tysiący dziesięćset siedemdziesiąciu sześciu]
   Mary wonders SE WH 1000.GEN 900.ACC 70.ACC 6.ACC
   studentów Jan zaprosił na imprezę.
   students.GEN John invited to party

b. Marija se pita [koliko hiljada devetsto sedamdeset šest] studenata je Ivan
   Mary SE wonders WH 1000.PL 900 70 6 students is Ivan
   pozvao na zabavu.
   invited to party.

b. Marija se pita [koliko hiljada devetsto sedamdeset šest] studenata je Ivan
   Mary SE wonders WH 1000.PL 900 70 6 students is Ivan
   pozvao na zabavu.
   invited to party.

Special numbers are not included in the wh-language.

d. Mary-wa John-ga paattii-ni [nan-zen kyuuhaku nanazuu roku]-nin-no
   MaryTOP JohnNOM party-LOC WH-1000 900 70 6 -CLS-GEN
   gakusei-o syootai-sita ka kango-teiru.
   studentACC invite-did Q wonder-ASP

(e. María se pregunta [qué mil novecientos setenta y seis] estudiantes invitó
   Maria SE wonders WH 1000 900 70 and 6 students invited
   Juan a la fiesta.
   Juan to the party.

Int. ‘Mary wonders which N John invited N-thousand nine hundred seventy six students.’

A cardinal in the end of a complex cardinal can also be replaced by a wh-item, as shown in (6).

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1 The sentences in (5) tend to be interpreted as an echo question.
(6) **Context:** One day, John told Mary that he invited 8976 students to their wedding party. Two weeks later, Mary forgot only the final digit of the number of the invited students.

a. *Maria zastanawia się [ osiem tysięcy dziewięćset siedemdziesiąciu ilu ]*
   
   Mary wonders SE 8.ACC 1000.GEN 900.ACC 70.ACC WH
   
   *students Jan zaprosił na imprezę.*
   
   [Polish]
   
   b. *Marija se pita [ osam hiljada devetsto sedamdest i koliko ] studenata je*
   
   Mary SE wonders 8 1000.PL 900 70 and WH students is
   
   *Ivan pozvao na zabavu.*
   
   [BCS]
   
   c. *Mary xiangzhidao John yaoqing [ ba-qian jiubai qishi ji ]-ge xuesheng lai*
   
   Mary wonders John invited 8-1000 900 70 WH -CLS students to
   
   *paidui.*
   
   [Chinese]
   
   d.  *
   
   *Mary-wa John-ga paatii-ni [ has-sen kyuuhyaku nanazyuu nan ]-nin-no*
   
   MaryTOP JohnNOM party-LOC 8-1000 900 70 WH -CLS-GEN
   
   *gakusei-o syootai-sita ka kangae-teiru.*
   
   [Japanese]
   
   e. *Maria se pregunta [ ocho mil novecientos setenta y qué ] estudiantes*
   
   Mary SE wonders 8 1000 900 70 and WH students
   
   *invitó Juan a la fiesta.*
   
   [Spanish]
   
   ‘Mary wonders which N John invited N-thousand nine hundred seventy six students.’

These data show that we can question a part of a complex cardinal by using a wh-item. However, it is important to notice that we cannot question only a multiplicand while keeping its multiplier intact. Let us now consider a different context. John told Mary that he invited 8976 students to their wedding party, and two weeks later Mary forgot the exact number of the invited students. However, she can remember that there are thousands of invited students, and that one digit of the number is 8. Now, she is wondering which multiplicand has 8 as its multiplier. In this situation, the relevant alternative set is something like: {8xxx students, x8xx students, xx8x students, xxx8 students}. Given that cardinals used as a multiplier can be replaced by a wh-item as shown in (5) and (6), the unified analysis predicts that we can question a multiplicand as well. This is because there is no semantic or syntactic difference between them under the unified analysis. However, this prediction is not borne out. We cannot question a multiplicand by using a wh-item. Again, it is not clear how to capture this contrast between multipliers and multiplicands under the unified analysis.

### 4. Proposal

I propose that a multiplicand is the head of UnitP, whereas a multiplier appears in Spec,UnitP. Unit\(^0\) can take a head noun or a silent numerical noun (NUMER) as its complement. Under my analysis, a complex cardinal and a head noun has the structure (7b) or (7c).

(7) a. “three hundred student” [meta-language]  
   
   ![Diagram](image)

   b. UnitP  
   
   “three”  
   
   Unit’  
   
   ![Diagram](image)  
   
   c. UnitP  
   
   “three”  
   
   Unit’  
   
   ![Diagram](image)
Based on Krifka’s (1995) analysis of classifiers, I propose that Unit\(^\circ\) has the denotation in (8). Unit\(^\circ\) contains the measure function \(\mu\), which takes a plural individual and returns the cardinality of that plural individual based on a given granularity level (cf. Solt 2014).

\[
(8) \quad \text{Unit}^{\circ} = \lambda P \in D_{<e,t>} \cdot \lambda y \in D_e \cdot \lambda x \in D_e \cdot [P(x) \land \mu_{\text{unit}}(x) = y]
\]

If an overt multiplicand appears in Unit\(^\circ\), a granularity level of the measure function \(\mu\) is specified according to the multiplicand. (7b,c) result in a complex cardinal when there is an overt multiplicand. If there is no overt multiplicand in Unit\(^\circ\), a granularity level of \(\mu\) is specified as 1; a contextually determined atomic entity (cf. Chierchia (2010)). The semantic computation of (7b) is given in (9).

\[
(9) \quad \text{a.} \quad \text{Unit}^{\circ} \text{[hundred]} = \lambda P \in D_{<e,t>} \cdot \lambda y \in D_e \cdot \lambda x \in D_e \cdot [P(x) \land \mu_{100}(x) = y]
\]

\[
\text{b.} \quad \text{Unit}^{\circ} \text{[np student]} = \lambda P \in D_{<e,t>} \cdot \lambda y \in D_e \cdot [P(x) \land \mu_{\text{student}}(x) \land \mu_{100}(x) = y]
\]

\[
\text{c.} \quad \text{Unit}^{\circ} \text{[np three hundred np student]} = \lambda x \in D_e \cdot [\text{number}(x) \land \mu_{100}(x) = 3]
\]

In (9), Unit\(^\circ\) first combines with a head noun. The measure function \(\mu\) returns the cardinality of \(x\) by counting members of \(x\). Since a granularity level is specified as 100, the object being counted in (9) should consist of 100 individuals. The multiplier specifies the cardinality of \(x\) as 3. Given that each member of \(x\) contains 100 individuals, there are 300 individuals in total. In a nutshell, the present analysis makes use of the measure function \(\mu\) to do multiplication, instead of covers and \(\Pi\). The semantic computation of (7c) is given in (10).

\[
(10) \quad \text{a.} \quad \text{Unit}^{\circ} \text{[hundred]} = \lambda P \in D_{<e,t>} \cdot \lambda y \in D_e \cdot \lambda x \in D_e \cdot [P(x) \land \mu_{100}(x) = y]
\]

\[
\text{b.} \quad \text{Unit}^{\circ} \text{[np number]} = \lambda y \in D_e \cdot [\text{number}(x) \land \mu_{100}(x) = y]
\]

\[
\text{c.} \quad \text{Unit}^{\circ} \text{[np three number]} = \lambda x \in D_e \cdot [\text{number}(x) \land \mu_{100}(x) = 3]
\]

\[
\text{d.} \quad \text{np three number}[\text{np student}] = \lambda x \in D_e \cdot [\text{number}(x) \land \text{student}(x) \land \mu_{100}(x) = 3] \quad \text{(Predicate Modification)}
\]

In (10), Unit\(^\circ\) first combines with the silent \text{number}. I assume that the silent \text{number} in (10) is interpreted as a property of being a number. Suppose that properties derived from a countable noun denote a semi-lattice structure (cf. Chierchia (1998), Bale & Barner (2009)), and that elements in a semi-lattice structure can be counted. Given this, it is not unreasonable to say that a property denoted by a countable noun can also be treated as a property of being a number. In (10), Unit\(^\circ\) takes the silent \text{number} as its complement, and then the multiplier is introduced. The UnitP combined with a noun phrase via Predicate Modification (Heim & Kratzer (1998)). Since properties derived from a countable noun can be mapped onto a property of being a number, the silent \text{number} in (10d) is semantically vacuous. This means that (9c) and (10d) are almost equivalent in semantics.

5. Support

5.1. Left-branch extraction

The example (4) can be captured under the proposed analysis. In (7c), a multiplier and a multiplicand form a constituent, excluding the head noun. The UnitP in (7c) can undergo LBE. In addition, the proposed analysis can capture other examples of LBE. As shown in (11a), a multiplier can undergo LBE while leaving a multiplicand in situ. In contrast, LBE of a multiplicand alone is disallowed, as in (11b).

\[\text{If the numeral “one” is used as a multiplier with an overt multiplicand, 1-deletion may take place in some languages (Hurford 1987, 2003).}\]
The contrast in (11) is not expected under the unified analysis. Moreover, the contrast cannot be captured even if we assume that a complex cardinal has the structure (12), which is a modified version of Ionin & Matushansky’s unified analysis. In (12), a multiplier and a multiplicand are analyzed as a phrasal element, rather than a head.

5.2. Wh-questions

Under the proposed analysis, the contrast between multipliers and multiplicands regarding wh-questions can also be captured. I proposed that a multiplicand specifies a granularity level of the measure function $\mu$, and does not denote a number. On the other hand, multipliers denote a number. Based on this, I suggest that the difference in wh-questions would reduce to the difference in the semantics of multipliers and multiplicands. We cannot question a multiplicand alone because languages we have seen so far do not have a wh-item that can be associated with a granularity level of the measure function. Remember that a number in the ones place can be questioned by a wh-item, as in (6). I propose that complex cardinals like “76” involve coordination of UnitPs, as shown in (15).
The number in the ones place is analyzed as the second UnitP. It can be questioned by replacing the whole second UnitP by a wh-item.\(^3\)

5.3. Russian approximate inversion

The proposal can be extended to Russian approximate inversion (AI), as in (16).

(16) a. \(\text{desjat’ kardinalov} \quad \text{[Russian]}\) ten cardinals

b. \(\text{kardinalov} \_ \text{desjat’} \Delta_1 \quad \text{[Russian]}\) cardinals ten

‘ten cardinals’

As shown in (17), when we apply AI to a noun modified by a complex cardinal, the multiplicand should move, instead of the modified noun phrase.

(17) a. \(\text{Van’a pr’iglas’il \, [tr’i sotn’i] \ stud’entov.} \quad \text{[Russian]}\)

‘Van invited three hundred students.’

b. \(\text{Van’a pr’iglas’il \, [sotn’i_1 tr’i} \_ \Delta_1 \ \text{stud’entov.} \quad \text{[Russian]}\)

‘Van invited approximately three hundred students.’

I suggest that Russian AI can be analyzed as movement of an overt head to a null functional head above the UnitP (cf. Franks (1995), Pereltsvaig (2006)). The structure is given in (18).

(18) a. \(\text{XP \, \text{UNIT P} \, \text{Unit’} \, \text{“three” \ UnitP \ “hundred” \ “student”}} \)

b. \(\text{XP \, \text{UNIT P} \, \text{Unit’} \, \text{“three” \ UnitP \ “student”}} \)

When there is an overt multiplicand as in (18a), the multiplicand is the closest overt head in the c-command domain of a null functional head that triggers AI. Therefore, the overt multiplicand must move to the higher functional head. In contrast, when Unit\(^0\) is null as in (18b), a head noun moves to the higher

\(^3\) Note that there is a variation among languages with respect to the availability of wh-questions of a multiplier. For instance, German never allows wh-questions of a multiplier. In Japanese and Chinese, it is possible to question a multiplier in any position in a complex cardinal.
functional head because it is the only overt head in the UnitP. It is worth noting here that Khrizman & Rothstein (2015) argue that a noun which undergoes approximate inversion exhibit the same behavior as a measure phrase. For example, a measure phrase can also undergo approximate inversion, as in (19).

(19) a. *pjat’ litrov moloka
   five liters milk
   ‘five liters of milk’

b. *litrov$_1$ pjat’ $\Delta_1$ moloka
   liters five milk
   ‘about five liters of milk’
   (Khrizman & Rothstein 2013: 261)

Furthermore, numerals require inanimate morphological case under a measure interpretation of nouns, as in (20a). The same pattern is observed with approximate inversion as in (20b).

(20) a. siloj rovno v {tri medvedje | *trex medvedej} [Russian]
   strength exactly in 3.ACC=NOM.INAN1 bear | 3.ACC=GEN.ANI bear
   ‘as strong as exactly three bears’

b. ja videl {soldata četyre | *soldat četyrex}. [Russian]
   I saw soldier.SG 4.ACC=NOM.INAN1 | soldier.PL 4.ACC=GEN.ANI
   ‘I saw about four soldiers.’
   (Khrizman & Rothstein 2013: 265-266)

These similarities between measure phrases and inverted nouns in AI may support the proposed analysis. Since the semantic denotation of Unit$_0$ includes the measurement function, it can be analyzed as a measure head discussed by Rothstein (2009). The inverted noun in AI moves through Unit$_0$ on the way to X under the proposed analysis, and it behaves like a measure phrase. In other words, an inverted multiplicand in (17b) is similar to litrov ‘liters’ in (19b) from syntactic and semantic point of views.

5.4. Japanese nominal ellipsis

As mentioned above, the proposed analysis treats Unit$_0$ (i.e. multiplicands) like a measure phrase in the sense that it contains a measure function associated with a specific granularity level. Another piece of evidence in favor of this analysis comes from nominal ellipsis in Japanese. Since the semantic denotation of Unit$_0$ includes the measurement function, it can be analyzed as a measure head discussed by Rothstein (2009). The inverted noun in AI moves through Unit$_0$ on the way to X under the proposed analysis, and it behaves like a measure phrase. In other words, an inverted multiplicand in (17b) is similar to litrov ‘liters’ in (19b) from syntactic and semantic point of views.

John-NOM 2-10-CLS-NO student -DAT meet-PAST but
‘Although John met 20 students, ...’

a. Mary-TOP 3-10-CLS-NO student -DAT meet-PAST
   ‘Mary met 30 students.’

b. *Mary-TOP 3-10-CLS-NO student -DAT mea-PAST
   ‘Mary met 30 students.’

c. Mary-TOP 3-10-CLS student -DAT meet-PAST
   ‘Mary met 30 students.’

d. ?Mary-TOP 3-10 student -DAT meet-PAST
   ‘Mary met 30 students.’

Saito & Murasugi (1990) argue that adjunct phrases cannot license nominal ellipsis because they do not trigger Spec-Head agreement. (21a) is a non-elliptical example. As shown in (21b), Japanese pre-nominal numeral classifiers do not license ellipsis. Based on this, Saito & Murasugi (1990) concluded that Japanese pre-nominal numeral classifiers are adjuncts.

(21) John-ga [ni-zyuu-nin-no gakusei] -ni at-ta kedo, ...
   John-NOM 2-10-CLS-NO student -DAT meet-PAST but
   ‘Although John met 20 students, ...’

a. Mary-TOP 3-10-CLS-NO student -DAT meet-PAST
   ‘Mary met 30 students.’

b. *Mary-TOP 3-10-CLS-NO student -DAT meet-PAST
   ‘Mary met 30 students.’

c. Mary-TOP 3-10-CLS student -DAT meet-PAST
   ‘Mary met 30 students.’

d. ?Mary-TOP 3-10 student -DAT meet-PAST
   ‘Mary met 30 students.’
In Japanese, a numeral classifier can be used as an argument without its head noun, as in (21c). Of importance here is that the numeral classifier in (21c) must be interpreted as ‘30 students’ even though there is no overt head noun. For example, (21c) is infelicitous in a situation where Mary met 30 teachers. The same is true for a complex cardinal without a classifier, as in (21d). (21d) is felicitous only if Mary met 30 students. Given these data, I assume that the numeral classifier in (21c) and the complex cardinal in (21d) are derived via nominal ellipsis. Their structures are given in (22).

\[
\text{(22)} \quad \begin{align*}
\text{a.} & \quad \text{UNITP} \\
& \quad \text{Unit}' \\
& \quad \text{“3”} \\
& \quad \text{NP} \\
& \quad \text{“student”} \\
& \quad \text{NUMBER} \\
& \quad “10” \\
\text{b.} & \quad \text{UNITP} \\
& \quad \text{Unit}' \\
& \quad “3” \\
& \quad \text{NP} \\
& \quad \text{“student”} \\
& \quad \text{NUMBER} \\
& \quad “10” \\
\end{align*}
\]

I assume that Unit° can be realized as a classifier in classifier languages such as Chinese and Japanese. Remember that the semantic denotation of Unit° given in (8) is adopted from Krifka’s analysis of classifiers. The denotation of the classifier in (21) is given in (23).

\[
\text{(23)} \quad \left[ \text{nin} \right] = \lambda P \in D_{<e,i>} . \lambda y \in D_e . \lambda x \in D_e . [P(x) \land \mu_{\text{person}}(x) = y]
\]

In (23), the granularity level of the measure function \( \mu \) is specified as a contextually determined atomic person. In other words, Japanese classifiers are analyzed as a different instance of Unit°. (22a) is the structure for the numeral classifier in (21c). Here the complex cardinal is located in Spec,UnitP. Since the complex cardinal can trigger Spec-Head agreement, the head noun can undergo nominal ellipsis. (22b) is the structure for the complex cardinal in (21d). Again, nominal ellipsis is available because the multiplier in Spec,UnitP can trigger Spec-Head agreement in this structure. The proposed analysis can capture the data in (21), especially the acceptability of (21d).

6. Conclusion

This paper argued that multipliers and multiplicands have a different status in syntax and semantics, contra Ionin & Matushansky’s (2006) unified analysis. I proposed that a multiplicand is the head of UnitP, and can take the silent NUMBER as its complement. On the other hand, a multiplier is always located in Spec,UnitP. This non-uniform treatment of multipliers and multiplicands can capture the various properties of complex cardinals.

References


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4 It is impossible to overtly realize a head noun phrase in (21c,d). See Huang & Ochi (2014) for an analysis of why Japanese noun phrases cannot be realized in the complement position of a classifier head.


