Complex Cardinals of Approximation

Jon Ander Mendia

1. Introduction

A variety of languages allow constructions where non–numerical expressions combine with numbers in order to form complex cardinal numerals. In such constructions, instead of denoting a single value, the resulting number denotes a range of possible values. In English, for instance, the indefinite determiner some can combine both with nouns and with numbers, as exemplified by the following two examples.

(1) a. Someone came to the party
   b. Twenty–some people came to the party

What the examples in (1) have in common is that any of them can be used to convey ignorance or uncertainty: (1a) conveys that the speaker is ignorant or maybe uncertain about who exactly came to the party, whereas in the case of (1b) the speaker might be ignorant about how many people exactly came to the party. In both cases, some is used to convey the same kind of indeterminacy.

In Spanish, similar constructions are possible with a number of different quantifiers. For instance, expressions similar to (1b) above can be formed with algún, the counterpart of the epistemic indefinite some in English. But, unlike English, Spanish also allows the vague determiners poco ("few") and mucho ("many") to combine with numbers.

(2) a. Los zapatos costaron sesenta y algunos euros
   b. Los zapatos costaron sesenta y pocos euros
   c. Los zapatos costaron sesenta y muchos euros

All these constructions have in common that they denote a range of values. As a consequence, they convey ignorance or uncertainty about the exact quantity in question. For instance, if the speaker knew that the shoes cost between 61 and 63 euros, she could truthfully and felicitously use either (2a) or (2b), but not (2c). If, instead, the shoes cost between 66 and 68 euros, (2b) would be infelicitous. And in a situation where the speaker knew the exact price of the shoes, it would be infelicitous to use any of the sentences in (2) as an answer to a question like how much did you pay for the shoes? Thus, there seems to be a connection between the range of values that these numeral expressions denote and the inferences that speakers draw from them. In this respect, they resemble epistemic indefinites and vague quantifiers in that they are incompatible with full knowledge, and so they require of the speaker to be in a certain belief/knowledge state (see Alonso-Ovalle & Menéndez-Benito 2010 for discussion).

In this paper, I provide an analysis of these constructions, which I will refer to as Cardinals of Approximation (CAs). I will first introduce the main data on Spanish. Then, building on previous work by Hurford (1975), Ionin & Matushansky (2006) and Anderson (2015), I provide a syntactic and a semantic analysis of CAs.1

---

1 Jon Ander Mendia, UMass Amherst, jmendiaaldama@linguist.edu. I would like to thank the audiences at ConSOLE XXIV and WCCFL XXXIII.

1 Space limitations preclude me from discussing the ignorance/uncertainty component of CAs, which I leave for a further occasion.

2. Properties of CAs

In Spanish CAs can be construed using a variety of different indefinites. These are the epistemic indefinite alguien ("some"), and the nouns pico and tantos, both of which denote indeterminate quantities.

(3) Los zapatos costaron sesenta y { pico / tantos / algunos } euros
the shoes cost sixty and some euros
'The shoes cost sixty-some euros'

Unlike English, Spanish also allows CAs with the vague quantifiers muchos and pocos, the counterparts of English "many'' and "few'' respectively. The paraphrases in (4b) and (5b) illustrate that the upper and lower bounds are the same in these constructions, except that the relative upper bound (in (4a)) and lower bound (in (5a)) must be adjusted.

(4) a. Los zapatos costaron sesenta y pocos euros
the shoes cost sixty and few euros
'The shoes cost a little above sixty euros'
b. The shoes cost at least 61 euros and no more than 65 euros.

(5) a. Los zapatos costaron sesenta y muchos euros
the shoes cost sixty and many euros
'The shoes cost a little less than sixty euros'
b. The shoes cost at least 65 euros and no more than 69 euros.

For convenience, I will use the expression #Q to refer to the family of quantificational elements that can appear in Spanish CA constructions.

2.1. Syntactic distribution

In the general case, #Qs combine with numerals that can independently combine by addition with some other number. That is, if a number like "six" can combine with "ten" to form "sixteen", as in (6a), then a #Q can also combine with "ten" to form a CA. The same is also true of larger numbers, like "sixty-six" in (7).

(6) a. diez y seis
ten and six
'Sixteen'
b. diez y { pico / pocos / muchos }
ten and some few many

(7) a. ciento sesenta y seis
hundred sixty and six
'A hundred and sixty-six'
b. ciento y { pico / pocos / muchos }
hundred and some few many

There are two main restrictions that CAs are subject to. First, to be a complex number is a necessary condition to form CAs. As a consequence, CAs are not allowed with simplex numbers. In Spanish, for instance, the number quince ("fifteen") is not morphologically complex in the same way that dieciseis ("sixteen") is; it lacks the conjunctive particle y ("and"). In those cases where complex numerals cannot be construed, CAs are not allowed.
(8)  
  a. *quince y dos  
     fifteen and two  
     Intended: 'Seventeen'  
  b. *quince y { pico / pocos / muchos }  
     fifteen and some few many  

  The second restriction concerns word order: #Qs cannot precede the rest of the number. That is,  
  reversing the relative order of numbers and #Qs, as in (6b) for instance, is not possible.

(9)  
  *{ pico / pocos / muchos } y seis  
  some few many and six  
  Intended: 'algunos y seis'; 'pocos y seis'; 'muchos y seis'  

  Only some dialects of Spanish allow a strategy of this sort, usually by eliding the #Q itself: (10) is  
  true if the number is 25, 35, 45, 55, 65, 75, 85 or 95.2

(10)  
  t - i - cinco  
  and 5  
  'Some and five'  

  The meaning of sentences like (10) can be understood as the addition of the number 5 and a place-
  holder that stands for all the tuples of ten that could independently form a complex number with 5.  
  (So, even in these dialects, ticinco cannot denote the simplex numbers five or fifteen (cinco and quince  
  respectively). Similar and more common variants of (10) can be found also in Iberian Spanish where the  
  number itself is replaced by a #Q, as in ta-i-tantos, and even taimuchos, although these are certainly not  
  as productive as the cases discussed previously; for instance, expressions like *taicinco or *muchos y  
  cinco are plainly ungrammatical.  

  Constructions like (10) are not completely surprising, however, given that other languages do allow  
  them freely. In Japanese, for instance, CAs formed with the indeterminate pronoun nan ("what"), allow  
  word orders where the indefinite precedes the rest of the numeral.

(11)  
  nan - juu - nin - ka - ga  
  what 10 CL.human part  
  'some ten people'  

[Anderson 2015]

2.2. Semantic properties

  CAs in Spanish come with both an at least and an at most component (see Anderson (2015) for  
  English CAs like (1b) above).

(12)  
  a. Twenty-some people came to the party  
  b. If 19 people came, the sentence is false.  
  c. If 32 people came the sentence is false.  
  d. If between 21 and 29 people came, the sentence is true.

  Thus, CAs are not just like so-called vague or approximate numbers, in the sense of Krifka (2009),  
  since vague numbers do not truth-conditionally denote a lower bound in the same way that CAs do.  
  Instead, vague numerals are simply less stringent with the bounds denoted by the number: the sentences  
  some twenty people came or around twenty people came are true if only 19 people came to the party,  
  unlike the CA construction "twenty-some".  

  With respect to the denotation of CAs, I take the upper and lower bound of CAs formed with algún,  
  pico and tantos to be uncontroversial.

---

What matters to establish the bounds of CAs is the value of the number it combines with, together to the value of the base that the number system uses in the language. This is because the range of numbers that CAs are compatible with is determined by the base of the number. In Spanish, as in English, the number system is of base 10, and so the values #Qs take can only denote in the range [1,9]. This is not true of other languages, however. The western variety of Basque, for instance, employs a predominantly base 20 number system. Since the possible denotations are determined by the base, CAs in this dialect are constructed such that #Qs denote in the [1,19] range.

Intuitions about the bounds of CAs formed with pocos and muchos in Spanish are, unfortunately, not as sharp. While all speakers admit that sentence (4a) is false if the shoes cost 69 euros, it is less clear where the exact upper bound of CAs formed with pocos ("few") is. A similar observation holds of CAs formed with the #Q muchos ("many").

A similar kind of complication arises with CAs denoting higher numbers. Take a numeral equivalent to "a hundred and few euros". Would the sentence be true in case the real value was 140? And 149? These are, ultimately, empirical questions that I will leave open here. For the purposes of this paper, I adopt the simplifying assumption that CAs with the #Qs pocos and muchos denote an upper and lower bound, respectively, that lies directly below (for pocos) and above (for muchos) the median of all the possible values that could be used instead of the #Q.

Unlike English, Spanish can form CAs with a variety of quantificational elements. Syntactically, #Qs can only combine with numbers additively, and so CAs are never found with simplex numbers that cannot form the first term of the an additive number (e.g., five, seven, etc.). In addition, CAs in Spanish cannot be pre-numeral (although there are certain non-productive constructions that do seem to be compatible with pre-numeral positions). In other languages, both pre- and post-numeral #Qs are allowed.

Semantically, CAs denote upper and lower bounds and so they are truth-conditionally compatible with a range of values. In turn, the relevant range is established depending on the base of the number system used in the language.

In the remainder of the paper I propose an analysis of CAs in Spanish by building on Alonso-Ovalle & Menéndez-Benito (2010) and Anderson (2015). The analysis I present aims to be general enough so as to be extensible to the wide variety of CA constructions in other languages.

2.3. Interim summary

Unlike English, Spanish can form CAs with a variety of quantificational elements. Syntactically, #Qs can only combine with numbers additively, and so CAs are never found with simplex numbers that cannot form the first term of the an additive number (e.g., five, seven, etc.). In addition, CAs in Spanish cannot be pre-numeral (although there are certain non-productive constructions that do seem to be compatible with pre-numeral positions). In other languages, both pre- and post-numeral #Qs are allowed.

Semantically, CAs denote upper and lower bounds and so they are truth-conditionally compatible with a range of values. In turn, the relevant range is established depending on the base of the number system used in the language.

In the remainder of the paper I propose an analysis of CAs in Spanish by building on Alonso-Ovalle & Menéndez-Benito (2010) and Anderson (2015). The analysis I present aims to be general enough so as to be extensible to the wide variety of CA constructions in other languages.

2. Background

I adopt a general view of the syntax-semantics of complex numeral constructions based on ideas present in Ionin & Matushansky (2006) and Solt (2015). The general geometry of DPs modified by
numerals is represented below in (17). The numerals themselves sit in the specifier position of a Measure Phrase headed by Meas, which combines directly with an NP.

![Image](17) 

Meas is similar to a gradable predicate in that it denotes a relation between a property of individuals, the NP, and a degree along some scale; in this case, the scale is set to cardinalities.

![Image](18) 

Simple and complex numbers, in turn, denote properties of degrees (Landman 2004).

![Image](19) 

Notice that under standard assumptions, the denotation of the NP is a property of individuals, type ⟨et⟩, and so it cannot combine with Meas directly, of type ⟨e; dt⟩. There are a number of ways to solve this compositional problem. Here I follow Solt (2015) and introduce a new mode of composition, Degree Argument Introduction (which is reminiscent of the "Restrict" rule in Chung & Ladusaw 2004).

![Image](20) 

With these assumptions, the semantic composition of a simple DP like sixty people is as follows.

![Image](21) 

In the case of complex additive cardinals, like sixty-two, I follow Ionin & Matushansky (2006) and assume that they are syntactically complex, built by coordinating different numbers. Concretely, I follow the formulation of this idea spelled-out in Anderson (2015), and assume the existence of an additive head Add that carries on the addition.

![Image](22) 

The assumed structure and interpretation of a complex cardinal like sixty-two is represented below.

![Image](23) 

In short, the task of Add is to take two singleton properties of degrees and to add them up, resulting in the property that is true of the degree that is equal to the sum. In the case of (24), the predicate is satisfied by degrees that are equal to the sum of 60 and 2.

4. Analysis

4.1. The syntax & semantics of complex numerals

So far, we have seen how to compute the semantic interpretation of complex additive numerals. However, as Hurford (1975) observed, numbers may combine in modes other than addition in order
to construct complex numerals. In English, for example, they can combine by addition and by multiplication.

\[(25) \quad 3869 = [(3 \times 1000) + [(6 \times 1000) + [(8 \times 10) + 9]]]\]

I will take Hurford’s (1975) observation seriously and assume that this schema reflects the actual LF structure of complex numerals. Thus, in addition to the additive head Add, I suggest to incorporate the geometry in (25) by introducing an additional head Mul, which works fully parallel to Add, with the exception that the operation it carries out is a multiplication, instead of addition.

\[(26) \quad \text{[Mul]} = \lambda D' \lambda D'' \lambda d . \exists d'd''[d = d' \times d'' \land D'(d') \land D''(d'')]

I suggest that the role of Mul, however, is more general. I propose that all numbers are internally computed as summarized in (30).

\[(27) \quad [B^i] = \lambda d.[d = 10^i]\]

The idea is to construct an LF that resembles the positional notation for numbers, such that the number 60 can be decomposed in $6 \times 3^1$, which corresponds to $6 \times 10$. The multiplication is carried out by the head Mul and since different base values are freely eligible by Mul, cardinals are always assigned a complex structure. For example, the number 3000 has a syntactic structure like $[3[\text{Mul } B^3]]$, which is then interpreted as $3 \times B^3$, denoting the property of degrees that are equal to 3000, $\lambda d.[d = 3000]$. Since complex additive cardinals must also combine by addition, the proposed structure for a complex additive numeral like 3689 is the following:

\[(28) \quad [[3 \text{ Mul } B^3][\text{Add } [6 \text{ Mul } B^2][\text{Add } [8 \text{ Mul } B^1][\text{Add } [9 \text{ Mul } B^0]]]]]

We can now derive the denotation of any complex number by recursively combining multiplicatively with bases and additively with other numbers. In the case of (28), for instance, each of the multiplicative numbers starts off from a bare number and a base, as in (29) for the number 3000. The denotation is then computed as summarized in (30).

\[(29) \quad \begin{align*}
\text{a.} \quad & [3] = \lambda d.[d = 3] \\
\text{b.} \quad & [B^3] = [[1000]] = \lambda d.[d = 1000]
\end{align*}

\[(30) \quad \begin{align*}
&[[3000] = [3[\text{Mul } B^3]] = [3[\text{Mul } 1000]] = \text{[Mul]}([[[1000]])([3]) \\
&= \lambda D' \lambda D'' \lambda d . \exists d'd''[d = d' \times d'' \land D'(d') \land D''(d'')[[[1000]])([3)] \\
&= \lambda d . \exists d'd''[d = d' \times d'' \land d' = 1000 \land d'' = 3]
\end{align*}

All these steps are necessary for each application of Mul. When numbers combine additively, a completely parallel process obtains. This is illustrated in (31) and (32) below.

\[(31) \quad \begin{align*}
\text{a.} \quad & [80] = \lambda d.[d = 80] \\
\text{b.} \quad & [9] = \lambda d.[d = 9]
\end{align*}

\[(32) \quad \begin{align*}
&[[89] = [80[\text{Add } 9]] = [[\text{Add}][[9]][[[80]) \\
&= \lambda D' \lambda D'' \lambda d . \exists d'd''[d = d' + d'' \land D'(d') \land D''(d'')[[9]][[[80]) \\
&= \lambda d . \exists d'd''[d = d' + d'' \land d' = 9 \land d'' = 80]
\end{align*}

Thus, after recursively applying the two operations, we reach the final denotation of 3689, which amounts, once more, to a property of degrees, $\lambda d.[d = 3689]$. 

4.2. The semantics of CAs

The gist of the analysis is to treat CAs as a kind of indefinite numerals, so that we can keep the parallelism with the quantifiers some and many/few. Thus, if they are to be treated as quantifiers, they must take a property as their first argument. With the current syntax, however, this is not possible. Following Kayne (2005) and Zweig (2005), I adopt the idea that this first NP complement of #Qs is a silent noun number. The denotation of number is simply the set of simplex degrees, those which obtain by combining with the smallest base $B^0$.

(33) a. few books = [[few number] books]\[Kayne 2005; Zweig 2005]\n    b. twenty-some books = [[twenty [some number]] books]\[Anderson 2015]\n
(34) \[
\text{[number]} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}
\]

Then, I adopt the definition of the #Qs algún, tantos and pico suggested by Anderson (2015) for English some (cf. Weir 2012). According to (35), these #Qs denote a subset selection function $f$ over degrees that selects some integer from the set of numbers denoted by its second argument. Crucially, $f$ comes with an anti-singleton presupposition, just like it has been proposed for algún by Alonso-Ovalle & Menéndez-Benito (2010).

(35) \[
\text{[pico]} = \lambda D (d) \lambda d. \text{anti-singleton}(f).[f(D)(d)]
\]

Instead, muchos and pocos are defined following their usual definition (under their cardinal variant in the GQT tradition; Barwise & Cooper 1981), whereby they determine the property of degrees that remain above/below a certain threshold. Concretely, I propose to define the meaning of algunos so that it further restricts the lowest possible value of $f$. This is done by introducing the median $M$ of some set of numbers $D$ provided by number. In the decimal system, the value of $M(D)$ will always be 5. Similarly, pocos is defined so that $f$ picks values in the range $[1, 4]$.

(36) \[
\text{[múchos]} = \lambda D \lambda d.([D](d) \land d > M(D))
\]

(37) \[
\text{[pocos]} = \lambda D \lambda d.([D](d) \land d < M(D))
\]

Under these assumptions, the LF structure of a CA like twenty-#Q is the following.

(38) \[
[\text{NumP}_1 20 \text{[Num}'] \text{add } [\text{NumP}_2 \text{[Num}'] \text{#Q number }] \text{mul } B^0 ]
\]

The derivation of CAs in Spanish (with the pico/algun/tanto and the mucho and poco types) proceeds alike. All three kinds of #Qs take the null NP number as argument and return a property of degrees.

(39) \[
[\text{NumP}] = [\#Q](\text{[number]})
\]

(40) a. \[
\text{[pico number]} = \lambda D \lambda d. \text{anti-singleton}(f).[[f(\langle D \rangle)(d))(\lambda d'[0 < d' < 10])
\]
    = \lambda d. [f(0 < d < 10)]
    = 1 \text{ iff } d \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}

b. \[
\text{[pocos number]} = \lambda D \lambda d.([D](d) \land d < M(D)))(\lambda d'[0 < d' < 10])
\]
    = \lambda d.[0 < d < 10 \land d < 5]
    = 1 \text{ iff } d \in \{1, 2, 3, 4\}

c. \[
\text{[múchos number]} = \lambda D \lambda d.([D](d) \land d > M(D))(\lambda d'[0 < d' < 10])
\]
    = \lambda d.[0 < d < 10 \land d > 5]
    = 1 \text{ iff } d \in \{6, 7, 8, 9\}

The resulting property of degrees is then combined with Mul, just as if it were a regular cardinal numeral.
(41) \[ \text{[NumP2]} = [\text{Mul}]([[B^0]])([[\text{NumP}^*]]) \\
= \lambda D' \lambda D'' \lambda d. \exists d''''[d = d' \times d'' \land D'(d') \land D''(d'')](\lambda d. [d = 1])([[\text{NumP}^*]]) \\
= \lambda d. \exists d''''[d = d' \times d'' \land d' = 1 \land [[\text{NumP}^*]](d'')] \]

(42) a. \[ [[\text{pico Mul } B^0]] = \lambda d. \exists d''''[d = d' \times d'' \land d' = 1 \land f(0 < d'' < 10)] \]

b. \[ [[\text{pocos Mul } B^0]] = \lambda d. \exists d''''[d = d' \times d'' \land d = 1 \land d'' < 5] \]

c. \[ [[\text{muchos Mul } B^0]] = \lambda d. \exists d''''[d = d' \times d'' \land d = 1 \land d'' > 5] \]

After the final step, the resulting NumP denotes a property of degrees, which can in turn serve as the first argument to Add. The resulting object is a property of degrees in each case.

(43) \[ [[[\text{veinte } #Q]] = \lambda D' \lambda D'' \lambda d. \exists d''''[d = d' + d'' \land D'(d') \land D''(d'')][[[#Q]]][[[20]]] \]

5. Conclusions

The analysis of CAs presented in this paper builds on previous work by Ionin & Matushansky (2006), Alonso-Ovalle & Menéndez-Benito (2010) and Anderson (2015) and aims to maintain the parallelism between CAs and other constructions involving indefinite quantifiers. The analysis defended here is flexible enough to account for why in languages with non-10 based numeral systems the denotations compatible with CAs vary accordingly with the base (as shown above for Basque). In addition, the system can also accommodate cases where CAs are formed with #Qs that precede the numeral expression (as in some dialects of Spanish). Lastly, the proposal can account for numerals where more than one #Q participate in the same CA construction, as it is possible to do in Japanese:

(44) \text{nan - zen} \quad \text{nan - byaky} \quad \text{nin} \quad \text{ka - ga} \quad
\text{what} \quad \text{thousand} \quad \text{what} \quad \text{hundred} \quad \text{CL-human} \quad \text{part} \quad \text{nom} \quad
\text{'Some-thousand-some-hundred people'}

Further research is required (i) to decide whether the LF representations of cardinal numerals proposed in this paper can be reduced to simpler structures in such a way that its semantic merits can be preserved, and (ii) to determine what factors determine the observed cross-linguistic variability and by what means.

References
