LITTLE: Not a Dichotomy-Based Negation Operator, but a Trivalence-Based Polar Opposition Operator

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1. Introduction

Few and *little* have two major uses: (1) illustrates their use as scalar adjectives in absolute constructions, and (2) illustrates their use in comparatives.[1] Though apparently these two uses look different, we intuitively feel that there exists something fundamentally the same: the use of *few/little* brings the meaning of polar opposition. In (1), *few* and *little* express the opposite of positive values meant by *many* and *much*, while in (2), the use of *few/little* reverses the direction of inequality.

(1) *Few* and *little* used as scalar adjectives in **absolute constructions**: $\rightsquigarrow$ antonyms of *many/much*
 a. John read (so / very) **few** books. (vs. *John read (so / very)* ***many*** *books.*)
 b. John has (so / very) **little** money. (vs. *John has (so / very)* ***much*** *money.*)

(2) *Few* and *little* used in **comparatives**:
 a. John has **fewer** books than Mary does. (vs. *John has* ***more*** *books than Mary does.*)
 b. John is **less** tall than Mary is. (vs. *John is tall**er** than Mary is.*)

The semantics of polar opposition brought by the use of *few/little* is certainly reminiscent of set-theoretic negation. In fact, in the formal semantics literature, the idea that *few/little* works as or contains a (degree) negation operator LITTLE has been much entertained (see, e.g., Rullmann 1995, Heim 2006, Solt 2006, Büring 2007a,b). Moreover, it has been proposed that many negative scalar adjectives (e.g., *small*, *short*, *slow*) can be decomposed into two parts: (i) their positive antonym (e.g., *large*, *tall*, *fast*), and (ii) this negation operator LITTLE (see Büring 2007a,b).

However, in this paper, we argue that set-theoretic negation, which is based on **dichotomy**, cannot fully characterize the interpretation of polar opposition by human cognition. Instead, we propose that the semantics of polar opposition involves **a trivalent distinction**, and thus the semantics of LITTLE should be based on this trivalent distinction.

Therefore, as summarized in (3), our major claim is that in natural language, there are two different kinds of opposition operators: **a dichotomy-based operator** and **a trivalence-based operator**.

(3) Two kinds of opposition operators in natural language: $\neg$ vs. LITTLE
 a. $\neg$ (e.g., *not*, *no*): a set-theoretic negation operator based on **dichotomy**.
 $\neg$ (i) takes a set as its argument, and (ii) returns **the complement of the set**.
 b. LITTLE (e.g., *few*, *little*): a scale-based operator of polar opposition based on **trivalence**.
 LITTLE (i) takes a positive value on a scale, and (ii) returns the negative value that is **the inverse of the positive value** with regard to **the neutral value on the scale**.

In §2, we present empirical motivation: the existence of the neutral value on a scale challenges dichotomy-based negation analyses of LITTLE and calls for a trivalent analysis. We present our proposal

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[1] Throughout this paper, we assume that *few* and *little* only differ in that *few* is used along with plural count nouns and incompatible with mass nouns, while *little* is used along with mass nouns and incompatible with count nouns. This complementary distribution is orthogonal to our concern here, so we consider *few* and *little* semantically equivalent. Similarly, we consider *many* and *much* semantically equivalent in our discussion.

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in §3. After introducing the basics of interval arithmetic in §4, we spell out in §5 and §6 the formal details of trivalence-based LITTLE used in absolute constructions and comparatives. §7 concludes the paper.

2. The Neutral Value: the Challenge to Negation-Based Analyses of LITTLE

2.1. LITTLE *Analyzed as Dichotomy-Based Negation Operator*

To begin with, here we use Büring (2007a,b)'s analysis to illustrate negation-based theories of LITTLE.[2] As shown in (4), LITTLE is considered as equivalent to set-theoretic negation.

(4) LITTLE $\stackrel{\text{def}}{=} \lambda P.\neg P$ (Büring 2007a,b)

Based on (4), Büring (2007a,b) decomposes a negative adjective A^- into two components: (i) its positive antonym A^+, and (ii) negation operator LITTLE (see (5)).[3] Thus, essentially, A^+ denotes the set of degrees on scale A to which x reaches (i.e., the degrees that x has), and A^- denotes the set of degrees on scale A to which x does not reach (i.e., the degrees that x lacks, see Kennedy 1997, 2001). In other words, a negative adjective, such as *short*, is essentially analyzed as *not tall*.

(5) In absolute constructions:

a. Positive adjective A^+: $[\![A^+]\!]_{\langle e,dt\rangle} \stackrel{\text{def}}{=} \lambda x.(0, A(x)]$
(Here A is of type $\langle ed\rangle$: it maps an individual x to a degree on scale A.)

b. Negative adjective A^-: $[\![A^-]\!]_{\langle e,dt\rangle} \stackrel{\text{def}}{=} \lambda x.[\text{LITTLE}[[\![A^+]\!](x)]] = \lambda x.(A(x), +\infty)$

Similarly, as shown in (6), Büring (2007a,b) also decomposes *less* into two components: (i) comparative morpheme *er*, and (ii) negation operator LITTLE. Essentially, *er* denotes a relation between two entities A and B such that on scale f, the set of degrees that A reaches entails the set of degrees that B reaches. The use of *less* reverses the direction of inequality, as shown in (6b).

(6) In comparatives:

a. $[\![\text{er}]\!]_{\langle\langle e,dt\rangle,\langle e,et\rangle\rangle} \stackrel{\text{def}}{=} \lambda f_{\langle e,dt\rangle}.\lambda A_e.\lambda B_e.f(B) \supset f(A)$ $\rightsquigarrow$ B is f-er than A.

b. $[\![\text{less}]\!] \stackrel{\text{def}}{=} \lambda f.\lambda A.\lambda B.[\text{LITTLE}[[\![\text{er}]\!](f)(A)(B)]] = \lambda f.\lambda A.\lambda B.f(B) \subseteq f(A)$
$\rightsquigarrow$ B is not f-er than A.

2.2. The Challenge to Dichotomy-Based Theories of LITTLE

However, as already pointed out in Solt (2006)'s analysis of *few* and suggested in some discussions on 'extension gap' (e.g., Kennedy 1997), our intuitions for A^- and $\neg A^+$ do not seem to be the same.

More specifically, according to our intuition, 'being tall' certainly entails 'not being short', but 'not being short' does not entail 'being tall'; similarly, 'being short' entails 'not being tall', but 'not being tall' does not entail 'being short'. In other words, the interpretation of scalar antonyms in human language is not based on **dichotomy** between A^- and A^+, but rather on **a trivalent distinction** among A^-, A^+, and $\neg A^- \sqcap \neg A^+$. Data presented in (7) clearly support this trivalent distinction.

(7) neither A^+ nor A^-

a. To him, the remaining hours of darkness were **neither few nor many**; ...

b. John is **neither tall nor short**.

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[2] Our presentation follows Heim (2008)'s version and simplifies Büring (2007a,b)'s original implementation.

[3] The motivation for Büring (2007a,b)'s decompositional analysis of negative adjectives is from data of cross-polar (a)nomalies (see (ia) and (ib)), but it is still debatable whether this decompositional view of negative adjectives is both theoretically favorable and empirically well-motivated (see the discussion in Heim 2008). The motivation for Büring (2007a,b)'s decomposition is not central to our proposal here, and will not be discussed.

(i) a. *John is **taller** than Mary is **short**. cross-polar anomaly

b. John is **shorter** than Mary is **tall**. cross-polar nomaly

Strictly speaking, 'extension gap', which originates in a dichotomy-based analysis of meaning, is a misleading term for this middle (or neutral) part $\neg A^- \sqcap \neg A^+$. In dichotomy-based theories, 'positive extension' corresponds to the set of entities that make a predication true, while 'negative extension' corresponds to the set of entities that make a predication false. In addition, there may be an 'extension gap', which corresponds to the set of entities that the predication is neither true nor false in a particular context (see, e.g., Kennedy 2007). Evidently, within a dichotomy-based view of meaning, the existence of 'extension gap' is a supervaluation issue: it is rooted in some kind of uncertainty (i.e., lack of information), vagueness, or undefinedness in a given context.

However, the existence of this neutral part $\neg A^- \sqcap \neg A^+$ in the semantics of scalar antonyms is not a supervaluation issue. It is clear to our intuition that the existence of this neutral part $\neg A^- \sqcap \neg A^+$ can be independent of uncertainty, vagueness, or undefinedness. In a given context, even if we are absolutely sure how the range and boundaries of largeness and smallness are defined for a group of entities (e.g., a set of dresses of various sizes fabricated precisely according to a well-defined size chart), we can still entertain the existence of a gap between largeness and smallness: the set of those middle-sized dresses. In other words, $\neg A^- \sqcap \neg A^+$ is not a truth-value gap, but the intersection of two 'negative extensions', and our human cognitive system makes a trivalent distinction in processing scalar antonyms.

When we take a closer look at comparatives, as illustrated in (8), the same point can be made. According to the negation-based analysis of LITTLE shown in (6), (8a) is predicted to have the same meaning as (8b), but obviously, this prediction is not borne out. If John and Mary have exactly the same height in a given scenario, then (8a) is clearly false, while (8b) is true. More specifically, while (8a) entails (8b), (8b) does not entail (8a): (8b) includes the middle part, i.e., the difference between John's height and Mary's height is zero, while (8a) does not include this middle part.

(8) Mary is **taller** than John is. $\rightsquigarrow$ LITTLE changes the direction of inequality, but $[\![(8a)]\!] \neq [\![(8b)]\!]$.

a. Mary is **less tall** than John is. HEIGHT(Mary) $<$ HEIGHT(John)

b. Mary is **not taller** than John is. HEIGHT(Mary) $\leq$ HEIGHT(John)

In sum, for both absolute constructions and comparatives, the interpretation of polar opposition calls for a trivalent analysis. If LITTLE is a polar opposition operator, it should be a trivalance-based operator.

3. Proposal: A Trivalence-Based Operator of Polar Opposition

Figure 1 illustrates our central claim. There are two different kinds of opposition operators in natural language: **a dichotomy-based negation operator**, and a **trivalence-based operator of polar opposition**. Figure 1 shows that the model of negation contains only two parts: a set and its complement, while the model of polar opposition is based on a scale, and contains three parts: a positive value, its inverse, and the neutral part between them.

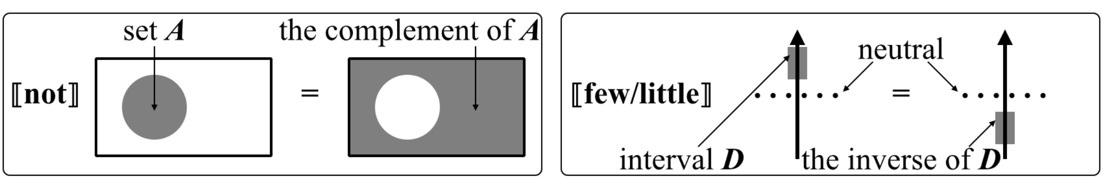

Figure 1: A dichotomy-based negation operator vs. a trivalence-based operator of polar opposition

Based on this distinction between negation and polar opposition, we propose that in addition to the negation operator $\neg$, there is a polar opposition operator LITTLE in natural language.

As shown in (9), LITTLE takes a positive value (here X^+) on a scale as its argument, and returns its **inverse** (i.e., a negative value, here X^-) with regard to the contextually given neutral part (here N).

(9) The definition of a trivalence-based operator of polar opposition – LITTLE:
$[\![X^-]\!] = \text{LITTLE}([\![X^+]\!]) = \iota z[X^+ - N = N - z]$

As illustrated in Figure 1, it is clear that the neutral part can be a range of degrees. Thus, we propose to implement our formalism with interval arithmetic. **Scales** are totally ordered sets of **degrees** (i.e., points) on different dimensions, and **intervals** are **convex subsets** of scales (see (10)).

(10) Definition of being a **convex** subset of a scale:
For a subset I (of a certain scale), it is a convex set (i.e., interval) iff for any two degrees d_1 and d_2, such that (i) $d_1 \in I \wedge d_2 \in I$ and (ii) $d_1 \leq d_2$, it follows that $\forall d[d_1 \leq d \leq d_2 \rightarrow d \in I]$.

Evidently, for natural language polar opposition, the neutral part as well as both the positive and negative parts can all be considered as convex sets, i.e., intervals, or a range of possible degree values.

Based on the concept of intervals, in the rest of this section, we informally analyze polar opposition involved in the semantics of scalar antonyms and comparatives.

3.1. Polar Opposition of Scalar Antonyms

From Figure 1, obviously, given a (context-dependent) neutral interval on a certain scale, the extension of a positive adjective on that scale can be considered as the interval above the neutral interval, and the extension of its negative antonym can be considered as the interval below the neutral interval (see (11)). Thus, when polar opposition operator LITTLE takes interval A^+ as its argument, it returns the inverse of A^+ with regard to the neutral interval, i.e., interval A^-.

(11) A trivalent distinction in the semantics of scalar antonyms:
a. $\neg A^+ \wedge \neg A^-$: the context-dependent neutral interval on a certain scale.
b. A^+: the interval **above** the neutral part on that scale.
c. A^-: the interval **below** the neutral part on that scale.

3.2. Polar Opposition in Comparatives

Here we propose a new way to look at polar opposition in comparatives. **The direction of inequality** in comparatives can be translated as **the polarity of the differential**. The differential is the result of subtracting the value of the comparative standard from the value of the comparative subject.

As shown in (12), the neutral case in comparatives is that there is no difference between the values of the comparative subject and the comparative standard on the relevant scale, or in other words, the differential is zero, which can be written as a singleton convex set $[0, 0]$.

Then obviously, when the differential $I_{\text{differential}}$ is above $[0, 0]$ (i.e., a positive value), the value of the comparative subject on the relevant scale exceeds the value of the comparative standard. When LITTLE takes this positive differential as its argument, it returns its inverse with regard to $[0, 0]$. The result is a negative differential, which means that the value of the comparative standard on the relevant scale exceeds the value of the comparative subject.

(12) A trivalent distinction in the semantics of comparatives:
$I_{\text{differential}}$ means 'Value of comparative subject B − Value of comparative standard A'.
a. There is no difference between B and A: $I_{\text{differential}} = [0, 0]$ $\rightsquigarrow$ the neutral interval.
b. B is f-er than A: $I_{\text{differential}} \subseteq (0, +\infty)$ $\rightsquigarrow$ $I_{\text{differential}}$ is **above** $[0, 0]$.
c. B is less f-er than A: $I_{\text{differential}} \subseteq (-\infty, 0)$ $\rightsquigarrow$ $I_{\text{differential}}$ is **below** $[0, 0]$.

In our analysis, the parallelism between polar opposition in comparatives and polar opposition of scalar antonyms is very evident. Thus, a unified account for the use of LITTLE in both constructions is conceivable: it always turns a positive value into its inverse with regard to the neutral value.

4. Interlude: Interval Arithmetic

To facilitate the presentation of our formal implementation in §5 and §6, we present the basics of interval arithmetic here. Those who are familiar with interval arithmetic can skip this section.

Since intervals are convex sets of degrees, as (13) shows, each interval can be written with its lower and upper bounds. Degrees are of type d; intervals are of type $\langle dt \rangle$.

(13) I: an interval of type $\langle dt \rangle$; its lower and upper bounds I_{min} and I_{max} are of type d.

a. $\{\delta | I_{\text{min}} \leq \delta \leq I_{\text{max}}\} = [I_{\text{min}}, I_{\text{max}}]$ Both the upper and lower bounds are closed.

b. $\{\delta | I_{\text{min}} < \delta \leq I_{\text{max}}\} = (I_{\text{min}}, I_{\text{max}}]$ Only the upper bound is closed.

c. $\{\delta | I_{\text{min}} \leq \delta < I_{\text{max}}\} = [I_{\text{min}}, I_{\text{max}})$ Only the lower bound is closed.

d. $\{\delta | I_{\text{min}} < \delta < I_{\text{max}}\} = (I_{\text{min}}, I_{\text{max}})$ Both the upper and lower bounds are open.

Since intervals represent ranges of possible degree values, as (14) shows, interval operations result in the largest possible range. Thus, (15) characterizes interval subtraction.

(14) Definition of interval operations: (See Moore 1979)
$[x_1, x_2]\langle\text{operator}\rangle[y_1, y_2] =$ the unique interval I such that

a. The lower bound of $I = \text{MIN}(x_1\langle\text{op}\rangle y_1, x_1\langle\text{op}\rangle y_2, x_2\langle\text{op}\rangle y_1, x_2\langle\text{op}\rangle y_2)$

b. The upper bound of $I = \text{MAX}(x_1\langle\text{op}\rangle y_1, x_1\langle\text{op}\rangle y_2, x_2\langle\text{op}\rangle y_1, x_2\langle\text{op}\rangle y_2)$

(15) Definition of interval subtraction:
$[x_1, x_2] - [y_1, y_2] = [x_1 - y_2, x_2 - y_1]$

Figure 2 illustrates two intervals A and B on a real number scale, and the subtraction between them is shown in (16).

(16) Examples of interval subtraction:

a. $B - A = [5, 8] - [2, 4] = [1, 6]$

b. $A - B = [2, 4] - [5, 8] = [-6, -1]$

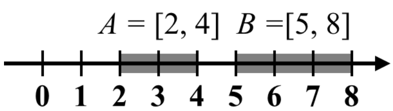

Figure 2: The subtraction between intervals A and B

Notice that interval subtraction is different from subtraction defined in number arithmetic: when X, Y, and Z are numbers, if $X - Y = Z$, it follows necessarily that $X - Z = Y$. However, when they represent intervals, as illustrated in (17), if $X - Y = Z$, in general, it does not follow that $X - Z = Y$. A consequence is that in interval arithmetic, given $X - Y = Z$ and the values of Y and Z, to compute the value of X, we must follow the recipe in (18).

(17) a. $[5, 8] - [2, 4] = [1, 6]$

b. $[5, 8] - [1, 6] = [-1, 7]$

(18) If $X - [a, b] = [c, d]$, generally speaking, it is NOT the case that $X = [a + c, b + d]$.

a. X is undefined if $b + c > a + d$, i.e., when X's lower bound is larger than its upper bound.

b. When defined, $X = [b + c, a + d]$.

5. Formal Analysis of Polar Opposition in Absolute Constructions

We implement all the positive, negative, and neutral values in terms of intervals on a scale.[4]

(19) is the definition of polar opposition operator LITTLE for intervals. It takes a positive value (i.e., an interval **above** I_{neutral}) as its argument, and it returns its inverse with regard to I_{neutral}.

(19) $\text{LITTLE}_{\langle dt,dt \rangle} \stackrel{\text{def}}{=} \lambda I_{\langle dt \rangle} : (I - I_{\text{neutral}}) \subseteq (0, +\infty).\iota I'[I - I_{\text{neutral}} = I_{\text{neutral}} - I']$ (= (9))

(20) shows that for absolute constructions in natural language, a positive value in a given context can be defined with the upper bound of the context-dependent neutral interval I^C_{neutral}.

(20) $\text{MANY}^C_{\langle dt \rangle} \stackrel{\text{def}}{=} \iota I[I - I^C_{\text{neutral}} = (0, +\infty)] = (I^C_{\text{neutral-upperbound}}, +\infty)$

[4] Our analyses in §5 and §6 only reflect (potentially silent) ingredients involved in lexical semantics of words. We stay neutral on the issue whether these ingredients are frozen at the word level or can affect syntactic derivations.

Based on (19) and (20), when LITTLE takes MANY^C as its argument, the result is the inverse of MANY^C with regard to the neutral interval (see (21)). Due to interval arithmetic, the meanings of **the upper and lower bounds of the neutral interval** contribute to the meanings of **the lower bound of the positive interval** and **the upper bound of its negative inverse** respectively (see (20) and (21)).

(21) $\text{LITTLE}\ [\text{MANY}^C] = \iota I'[I^C_{\text{neutral}} - I' = (0, +\infty)] = (-\infty, I^C_{\text{neutral-lowerbound}})$

Essentially, positive scalar adjectives relate an interval and an individual (see (22)): the value of the individual on a relevant scale is a subset of the interval. As (22)–(24) show, the relevant scale for *many/much* and *few/little* is amount. In absolute constructions, positive scalar adjectives contain a silent, contextually defined positive value MANY^C (see (23)), and their negative antonyms contain the inverse of that contextually defined positive value, which is $\text{LITTLE}\ [\text{MANY}^C]$ (see (24)).

(22) $[\![\text{many/much}]\!]_{\langle dt,et\rangle} \overset{\text{def}}{=} \lambda I_{\langle dt\rangle}.\lambda x_e.[\text{AMOUNT}(x) \subseteq I]$

(23) $[\![\text{many/much}_{\text{absolute}}]\!] = [\![\text{many/much}]\!]\ [\text{MANY}^C]$ $\quad$ MANY^C is silent.
$= \lambda x.[\text{AMOUNT}(x) \subseteq (I^C_{\text{neutral-upperbound}}, +\infty)]$

(24) $[\![\text{few/little}_{\text{absolute}}]\!] = [\![\text{many/much}]\!]\ [\text{LITTLE}\ [\text{MANY}^C]]$ $\quad$ MANY^C is silent.
$= \lambda x.[\text{AMOUNT}(x) \subseteq (-\infty, I^C_{\text{neutral-lowerbound}})]$

Other scalar adjectives can be analyzed in a similar way. (25)–(29) show the semantics of *tall* and *short*. (25) shows that *tall* relates the height of an individual and an interval on the height scale. In absolute constructions of *tall* and *short* (see (26) and (27)), the interval arguments of *tall*, i.e., MANY^C and $\text{LITTLE}\ [\text{MANY}^C]$, depend on contexts; while in measure construction (28), the interval argument is explicitly expressed, here *6 feet*. Since *short* already means 'tall $[\text{LITTLE}\ [\text{MANY}^C]]$', i.e., the interval argument slot of *tall* is already filled, *short* cannot be used in measure constructions (see (29)).

(25) $[\![\text{tall}]\!]_{\langle dt,et\rangle} \overset{\text{def}}{=} \lambda I_{\langle dt\rangle}.\lambda x_e.[\text{HEIGHT}(x) \subseteq I]$

(26) Mary is tall. $\quad$ Absolute construction
  a. LF: Mary is $[\text{tall MANY}^C]$ $\quad$ MANY^C is silent.
  b. $[\![(26)]\!] = \text{HEIGHT}(\text{Mary}) \subseteq (I^C_{\text{neutral-upperbound}}, +\infty)$

(27) Mary is short. $\quad$ Absolute construction
  a. LF: Mary is $[\text{tall}\ [\text{LITTLE}\ [\text{MANY}^C]]]$ $\quad$ MANY^C is silent.
  b. $[\![(27)]\!] = \text{HEIGHT}(\text{Mary}) \subseteq (-\infty, I^C_{\text{neutral-lowerbound}})$

(28) Mary is 6 feet tall. $\quad$ Measure construction
  a. LF: Mary is [tall 6-feet]
  b. $[\![(28)]\!] = \text{HEIGHT}(\text{Mary}) \subseteq [6', 6']$ $\quad$ (a pragmatically strengthened reading of *6 feet*)
  c. $[\![(28)]\!] = \text{HEIGHT}(\text{Mary}) \subseteq [6', +\infty)$

(29) *Mary is 6 feet short. $\quad$ ✗ Measure construction

6. Formal Analysis of *Few/Little* in Comparatives

Here we follow Zhang & Ling (2015)'s interval-based analysis of comparatives, which is summarized in (30) (see Schwarzchild & Wilkinson 2002 and Beck 2010 for other implementations of interval-based theories of comparatives). The lexical entries of *more/-er* and *than* are given in (31) and (32).

(30) The semantics of comparatives in Zhang & Ling (2015):
  a. Comparatives express the existence of a distance between the positions of two values (the value of the comparative standard and the value of the comparative subject) on a scale.
  In terms of intervals: $I_{\text{subject}} - I_{\text{standard}} = I_{\text{differential}}$ ($I_{\text{differential}} \neq [0, 0]$)
  b. Comparative morpheme *more/er* is the differential by default: an interval $I \subseteq (0, +\infty)$.

(31) $[\![\text{more/-er}]\!]_{\langle dt\rangle} \stackrel{\text{def}}{=} I$ such that $I \subseteq (0, +\infty)$

(32) $[\![\text{than}]\!]_{\langle dt,\langle dt,dt\rangle\rangle} \stackrel{\text{def}}{=} \lambda I_{\text{standard}}.\lambda I_{\text{differential}}.\iota I[I - I_{\text{standard}} = I_{\text{differential}}]$

The semantics of comparative standards is expressed by *than*-clauses. As (33) shows, there is (i) a lambda-abstraction over interval variables and (ii) the insertion of a silent *the*, which picks out the contextually most informative interval I that represents, e.g., Sue's height or the boys' height (see also Beck 2010). For example, if Sue is exactly 5 feet tall, $[\![\text{(than) Sue is (tall)}]\!] = [5', 5']$; if the shortest boys is exactly 5 feet tall, and the tallest boy is exactly 6 feet tall, then $[\![\text{(than) every boy is (tall)}]\!] = [5', 6']$. Of course, informativeness is context-dependent and depends especially on the precision of measurement.

(33) *Than*-clause: I_{standard}

a. $[\![\text{(than) Sue is (tall)}]\!] = [\![\text{the}]\!][\lambda I.\textsc{height}(\text{Sue}) \subseteq I]$
$= [I_{\textsc{height}(\text{Sue})\text{-lowerbound}}, I_{\textsc{height}(\text{Sue})\text{-upperbound}}]$ $\rightsquigarrow$ the height of Sue

b. $[\![\text{(than) every boy is (tall)}]\!] = [\![\text{the}]\!][\lambda I.[\forall x[\text{boy}(x) \rightarrow \textsc{height}(x) \subseteq I]]$
$= [I_{\textsc{height}(\text{boys})\text{-lowerbound}}, I_{\textsc{height}(\text{boys})\text{-upperbound}}]$ $\rightsquigarrow$ the height of boys

(34) illustrates how a basic comparative sentence is analyzed, and (35) illustrates how comparatives with overt differential expressions are analyzed. In (35), the overtly expressed differential *at most 3 inches* restricts the default differential *er* so that in this sentence, the differential is $(0, 3'']$ (see (35b)).

Notice that in (35), due to the recipe in (18), the **lower bound** of the comparative subject is composed of (i) the **upper bound** of the comparative standard, and (ii) the **lower bound** of the differential (here 0), while the **upper bound** of the comparative subject is composed of (i) the **lower bound** of the comparative standard, and (ii) the **upper bound** of the differential (here $+3''$). Thus, when the differential is upward-entailing (i.e., its upper bound is $+\infty$), the sentence has a so-called MAX-interpretation, i.e., only the upper bound of the comparative standard eventually matters (see (34d)). However, when the differential is not upward-entailing, both the upper and lower bounds of the comparative standard affect the interval representing the comparative subject, as shown in (35d). Moreover, in (35), for the sentence to be defined, it has to be the case that $I_{\textsc{height}(\text{boys})\text{-upperbound}} < I_{\textsc{height}(\text{boys})\text{-lowerbound}} + 3''$, i.e., the difference between the tallest and shortest boys is less than 3 inches.

(34) Mary is **taller** than Sue is.

a. LF: Mary is [TALL [er [than $\textsc{height}$(Sue)]]
b. $I_{\text{differential}} = [\![\text{er}]\!] = (0, +\infty)$
c. $[\![(34)]\!] \Leftrightarrow \textsc{height}(\text{Mary}) \subseteq \iota I[I - I_{\textsc{height}(\text{Sue})} = (0, +\infty)]$
d. After simplification (see (18)), $\textsc{height}(\text{Mary}) \subseteq (I_{\textsc{height}(\text{Sue})\text{-upperbound}}, +\infty)$

(35) Mary is **at most 3 inches taller** than every boy is.

a. LF: Mary is [TALL [at most 3 inches ... er [than $\textsc{height}$(boys)]]
b. $I_{\text{differential}} = [\![\text{at most 3 inches ... er}]\!] = (0, +\infty) \sqcap (-\infty, 3''] = (0, 3'']$
c. $[\![(35)]\!] \Leftrightarrow \textsc{height}(\text{Mary}) \subseteq \iota I[I - I_{\textsc{height}(\text{boys})} = (0, 3'']]$
d. $\therefore \textsc{height}(\text{Mary}) \subseteq (I_{\textsc{height}(\text{boys})\text{-upperbound}}, I_{\textsc{height}(\text{boys})\text{-lowerbound}} + 3'']$

As shown in (36), the use of polar opposition operator LITTLE in comparatives returns the inverse of the positive differential. (37), (38) and (39) illustrate how *less*-comparatives with various kinds of overt differential expressions are analyzed. In these cases, there are two steps in the computation of differentials: first, *er* is restricted by overtly expressed differentials; second, the use of LITTLE changes the polarity of the differential, and consequently, it shifts the direction of inequality.

(36) $[\![\text{fewer/less}]\!] = \textsc{little}\ [\![\text{er}]\!]$ I.e., a certain interval I such that $I \subseteq (-\infty, 0)$.

(37) Mary is **more than 3 inches less tall** than every boy is.

a. LF: Mary is [$\textsc{tall}$ [more than 3 inches ... less [than $\textsc{height}$(boys)]]

b. $I_{\text{differential}} = \textsc{little}[\![\text{more than 3 inches ... er}]\!] = \textsc{little}(3'', +\infty) = (-\infty, -3'')$

c. $[\![(37)]\!] \Leftrightarrow \textsc{height}(\text{Mary}) \subseteq \iota I[I - I_{\textsc{height}(\text{boys})} = (-\infty, -3'')]$

d. $\therefore \textsc{height}(\text{Mary}) \subseteq (-\infty, I_{\textsc{height}(\text{boys})\text{-lowerbound}} - 3'')$

(38) Mary is **at most 3 inches less tall** than every boy is.

a. LF: Mary is [$\textsc{tall}$ [at most 3 inches ... less [than $\textsc{height}$(boys)]]

b. $I_{\text{differential}} = \textsc{little}[\![\text{at most 3 inches ... er}]\!] = \textsc{little}(0, 3''] = [-3'', 0)$

c. $[\![(38)]\!] \Leftrightarrow \textsc{height}(\text{Mary}) \subseteq \iota I[I - I_{\textsc{height}(\text{boys})} = [-3'', 0)]$

d. $\therefore \textsc{height}(\text{Mary}) \subseteq [I_{\textsc{height}(\text{boys})\text{-upperbound}} - 3'', I_{\textsc{height}(\text{boys})\text{-lowerbound}})$

(39) Mary is **between 3 and 4 inches less tall** than every boy is.

a. LF: Mary is [$\textsc{tall}$ [between 3 and 4 inches ... less [than $\textsc{height}$(boys)]]

b. $I_{\text{differential}} = \textsc{little}[\![\text{between 3 and 4 inches ... er}]\!] = \textsc{little}[3'', 4''] = [-4'', -3'']$

c. $[\![(39)]\!] \Leftrightarrow \textsc{height}(\text{Mary}) \subseteq \iota I[I - I_{\textsc{height}(\text{boys})} = [-4'', -3'']]$

d. $\therefore \textsc{height}(\text{Mary}) \subseteq [I_{\textsc{height}(\text{boys})\text{-upperbound}} - 4'', I_{\textsc{height}(\text{boys})\text{-lowerbound}} - 3'']$

Our analysis can potentially shed light on several issues of comparatives. First, no matter whether LITTLE is used or not, due to (18), the *than*-clause is always a downward-entailing environment and licenses NPI. Second, for comparatives to make sense, the length of I_{standard} cannot exceed the length of $I_{\text{differential}}$. Third, it seems that there is no need for *than*-clause internal quantifier to take scope anyway.

7. Concluding Remarks

In this paper, we argue for a trivalence-based analysis for polar opposition in the study of natural language scalar adjectives. We implement our formalism in interval arithmetic. Moreover, by analyzing the direction of inequality in comparatives as the polarity of differentials, we provide a unified account for the use of LITTLE (or *few/little*) in both absolute constructions and comparatives.

Our analysis of polar opposition seems to have a few ingredients in common with the semantics of negative adjectives proposed in Sassoon (2010). According to Sassoon (2010), negative adjectives map entities to values that are **linearly reversed** and **linearly transformed** in comparison with their values in the positive antonyms. Linear reversal in Sassoon (2010)'s theory clearly corresponds to the generation of the inverse in our analysis, while linear transformation in Sassoon (2010)'s theory bears some resemblance with the role the neutral part plays in our account (see (9) or (19)). There is a major difference between our proposal and Sassoon (2010)'s analysis: for Sassoon (2010), the value used in linear transformation is unspecified, but in our analysis, it is either the context-dependent middle value in absolute constructions (e.g., the part corresponding to *neither many nor few*) or the singleton interval $[0, 0]$ in comparatives. It would be interesting to further investigate whether our current proposal inherits all the explanatory and predictive benefits of Sassoon (2010), and this is left for future research.

Another worthy follow-up is to rethink the issue of decomposing negative scalar adjectives and its potential consequences at the sentence level. We have been staying neutral on this issue in the current paper (see our Footnotes 3 and 4). Heim 2008 suggests that degree negation might be frozen in the lexical semantics of negative scalar adjectives (i.e., inactive at the sentential level), but then it seems that cross-polar nomalies cannot be accounted for. Whether shifting from degree negation to trivalence-based polar opposition can bring an empirical improvement on this issue is also left for future research.

References

Beck, Sigrid (2010). Quantifiers in *than*-clauses. *Semantics and Pragmatics* 3:1, 1–72.

Büring, Daniel (2007a). Cross-polar nomalies. Friedman, T. & M. Gibson (eds.), *Proceedings of Semantics and Linguistic Theory 17*, 37–52. http://dx.doi.org/10.3765/salt.v17i0.2957.

Büring, Daniel (2007b). *More* or *less*. *Proceedings of the 43th Annual Meeting of the Chicago Linguistic Society*, 3–17.

Heim, Irene (2006). *Little*. Gibson, Masayuki & Jonathan Howell (eds.), *Proceedings of Semantics and Linguistic Theory 16*, 35–58.

Heim, Irene (2008). Decomposing antonyms. Grønn, Atle (ed.), *Proceedings of Sinn und Bedeutung 12*, 212–225.

Kennedy, Christopher (1997). *Projecting the adjective: The syntax and semantics of gradability and comparison*. Ph.D. thesis, University of California, Santa Cruz.

Kennedy, Christopher (2001). Polar opposition and the ontology of 'degrees'. *Linguistics and Philosophy* 24:1, 33–70.

Kennedy, Christopher (2007). Vagueness and grammar: The semantics of relative and absolute gradable adjectives. *Linguistics and Philosophy* 30:1, 1–45.

Moore, Ramon E. (1979). *Methods and Applications of Interval Analysis*. SIAM.

Rullmann, Hotze (1995). *Maximality in the semantics of wh-constructions*. Ph.D. thesis, UMass Amherst.

Sassoon, Galit Weidman (2010). The degree functions of negative adjectives. *Natural Language Semantics* 18:2, 141–181.

Schwarzchild, Roger & Karina Wilkinson (2002). Quantifiers in comparatives: A semantics of degree based on intervals. *Natural Language Semantics* 10:1, 1–41.

Solt, Stephanie (2006). Monotonicity, Closure and the Semantics of *few*. Baumer, Donald, David Montero & Michael Scanlon (eds.), *Proceedings of the 25th West Coast Conference on Formal Linguistics*, Somerville, MA: Cascadilla Proceedings Project, 380 – 389. http://www.lingref.com/cpp/wccfl/25/paper1471.pdf.

Zhang, Linmin & Jia Ling (2015). Comparatives revisited: Downward-entailing differentials do not threaten encapsulation theories. Brochhagen, Thomas, Floris Roelofsen & Nadine Theiler (eds.), *Proceedings of the 20th Amsterdam Colloquium*, 478–487.

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