

reflexivity axiom (2). It says that each string is in correspondence with itself and can thus be interpreted as both an underlying and a surface form (for discussion, see Moreton 2004b and Magri 2015a).

- (2) If the candidate set contains a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ with a surface string \mathbf{b} , then it also contains the *identity* candidate $(\mathbf{b}, \mathbf{b}, \mathbb{I}_{\mathbf{b},\mathbf{b}})$, whose underlying and surface strings are both equal to \mathbf{b} and whose correspondence relation is the identity $\mathbb{I}_{\mathbf{b},\mathbf{b}}$ among the segments of \mathbf{b} .

Within the representational framework just defined, a phonological *grammar* is a map G which takes an underlying form \mathbf{a} and returns a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ whose underlying string is indeed \mathbf{a} .² A string \mathbf{b} is called *phonotactically licit* according to a grammar G provided there exists an underlying form \mathbf{a} such that the grammar G maps \mathbf{a} to a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ whose surface string is \mathbf{b} . A grammar G is *idempotent* provided it maps any phonotactically licit surface form to itself, as formalized by the implication (3) in the following definition. The antecedent of the implication says that the surface form \mathbf{b} is phonotactically licit relative to the grammar G , because it is the surface realization of some underlying form \mathbf{a} . The consequent says that \mathbf{b} is then mapped faithfully to itself. Axiom (2) ensures that the candidate set indeed contains the identity candidate $(\mathbf{b}, \mathbf{b}, \mathbb{I}_{\mathbf{b},\mathbf{b}})$ which appears in the consequent of (3).

Definition 1 (Idempotency) A grammar G is idempotent provided it satisfies the following implication

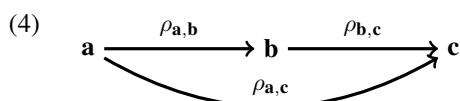
- (3) **If:** $G(\mathbf{a}) = (\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$
Then: $G(\mathbf{b}) = (\mathbf{b}, \mathbf{b}, \mathbb{I}_{\mathbf{b},\mathbf{b}})$

for any candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ in the candidate set. □

To illustrate, suppose that a grammar enforces final devoicing and thus maps the underlying form /rad/ to [rat]. The surface form [rat] is thus phonotactically licit. In order for that grammar to comply with condition (3) and thus qualify as idempotent, it needs to map the underlying form /rat/ faithfully to [rat].

2. First step: the faithfulness idempotency implication

Which conditions ensure that the the OT grammar corresponding to any ranking of a given constraint set is idempotent? Consider a *classical* version of OT, whereby constraints come in only two varieties (this is Moreton's 2004 *conservativity* assumption): *faithfulness* constraints, which assign no violations to any identity candidate; and *markedness* constraints, which are blind to underlying forms, namely assign the same number of violations to any two candidates sharing the surface form. Within this classical architecture, the following proposition 1 provides sufficient conditions for idempotency. The assumption made by the proposition is twofold. On the one hand, it restricts the candidate set: if it contains two candidates $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ and $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ which share a string \mathbf{b} as the underlying and surface form respectively, it must also contain a candidate $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{c}})$ which puts the underlying string \mathbf{a} of the former candidate in correspondence with the surface string \mathbf{c} of the latter candidate, as depicted in (4).



On the other hand, the assumption of the proposition restricts the constraint set: it requires each faithfulness constraint to satisfy the implication (5), which will be referred to as the *faithfulness idempotency implication* (FII).

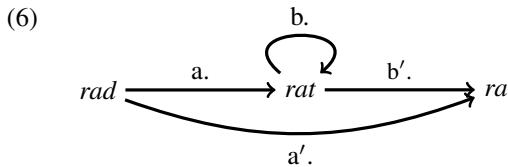
² For the sake of simplicity, the paper assumes that a grammar G maps an underlying form to a *single* candidate. This assumption is not crucial: the analyses developed here extend to a framework where G maps an underlying form to a *set* of candidates, thus allowing for phonological variation.

Proposition 1 Assume that, for any two candidates $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a}, \mathbf{b}})$ and $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b}, \mathbf{c}})$, the candidate set also contains a candidate $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a}, \mathbf{c}})$ such that the following implication holds for every faithfulness constraint F in the constraint set.

- (5) **If:** $F(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b}, \mathbf{c}}) = 0$
Then: $F(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a}, \mathbf{c}}) \leq F(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a}, \mathbf{b}})$

Then, the OT grammar corresponding to any ranking of the constraint set is idempotent. \square

Proposition 1 is derived in Magri (2015a:section 3), by mimicking an analogous proposition for output-drivenness obtained in Tesar (2013:chapter 3); see also Moreton & Smolensky (2002:section 3). Here is an informal derivation of the FII (5). Suppose that an OT grammar maps the underlying form */rad/* to the surface form [rat], as represented by the arrow (6a). This means that [rat] is phonotactically licit. Idempotency then requires the underlying form */rat/* to be faithfully mapped to [rat], as represented by the loop (6b). We reason by contradiction. Thus, we make the *contradictory assumption* that idempotency fails and that */rat/* is instead mapped to something else, say [ra] for concreteness, as represented by the arrow (6b'). In order to establish idempotency, we want to derive the *contradictory conclusion* that */rad/* is then mapped to [ra] as well, as represented by the arrow (6a'), against the hypothesis that */rad/* be mapped to [rat] that we started with.



To this end, assume that condition (7) holds for every constraint C in the constraint set. The antecedent says that C is “implicated” in the contradictory hypothesis that the mapping (6b') wins over the idempotent mapping (6b). The consequent says that C is then also “implicated” in the contradictory conclusion that the mapping (6a') wins over the attested mapping (6a). Thus, (7) is a sufficient condition for the contradictory assumption to entail the contradictory conclusion. In other words, (7) is a sufficient condition for the idempotent mapping of */rat/* to [rat].

- (7) **If:** constraint C doesn't prefer the idempotent mapping (6b) to the contradictory mapping (6b').
Then: constraint C doesn't prefer the actual mapping (6a) to the contradictory mapping (6a').

The mappings (6b)-(6b') compared in the antecedent of (7) feature the underlying form */rat/*. The mappings (6a)-(6a') compared in the consequent instead feature the underlying form */rad/*. The implication (7) thus trivially holds for the markedness constraints, because they are blind to the underlying forms, so that the antecedent and the consequent coincide. In conclusion, the implication (7) is a condition on the faithfulness constraints which illustrates the FII (5) in a concrete case.

Proposition 1 makes no assumptions on the nature of the correspondence relation $\rho_{\mathbf{a}, \mathbf{c}}$ depicted in (4) and in particular on its relationship with the two other correspondence relations $\rho_{\mathbf{a}, \mathbf{b}}$ and $\rho_{\mathbf{b}, \mathbf{c}}$. For instance, $\rho_{\mathbf{a}, \mathbf{c}}$ could be the empty relation. This would make the FII (5) trivial when F is an identity faithfulness constraint (because the quantity on the left-hand side of the inequality in the consequent would be equal to zero) but difficult when F is DEP or MAX (because the quantity on the left-hand side of the inequality would be large in this case). At the opposite extreme, $\rho_{\mathbf{a}, \mathbf{c}}$ could be the total relation, which puts any underlying segment in correspondence with any surface segment. This would make the FII (5) trivial when F is DEP or MAX but difficult when F is an identity faithfulness constraint. A natural assumption is that $\rho_{\mathbf{a}, \mathbf{c}}$ is the *composition* $\rho_{\mathbf{a}, \mathbf{c}} = \rho_{\mathbf{a}, \mathbf{b}} \rho_{\mathbf{b}, \mathbf{c}}$ of the two correspondence relations $\rho_{\mathbf{a}, \mathbf{b}}$ and $\rho_{\mathbf{b}, \mathbf{c}}$.³ This means that a segment a of the string \mathbf{a} and a segment c of the string \mathbf{c} are in correspondence

³ The operation of composition between two relations is usually denoted by “ \circ ”. In the rest of the paper, I write more succinctly $\rho_{\mathbf{a}, \mathbf{b}} \rho_{\mathbf{b}, \mathbf{c}}$ instead of $\rho_{\mathbf{a}, \mathbf{b}} \circ \rho_{\mathbf{b}, \mathbf{c}}$.

through $\rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}}$ if and only if there exists some “mediating” segment b of the string \mathbf{b} such that a is in correspondence with b through $\rho_{\mathbf{a},\mathbf{b}}$ and furthermore b is in correspondence with c through $\rho_{\mathbf{b},\mathbf{c}}$. The candidacy *transitivity axiom* (8) ensures the existence of this *composition candidate*, thus complementing the reflexivity axiom (2).

- (8) If the candidate set contains two candidates $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ and $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ which share a string \mathbf{b} as the underlying and surface form respectively, it also contains the *composition candidate* $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}})$ whose correspondence relation $\rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}}$ is the composition of $\rho_{\mathbf{a},\mathbf{b}}$ and $\rho_{\mathbf{b},\mathbf{c}}$.

The original FII (5) can now be specialized in terms of this composition candidate as the implication (9), which will be referred to as the FII_{comp} . The FII_{comp} entails the original FII and thus provides a sufficient condition for the idempotency of all the grammars in an OT typology.

$$(9) \quad \text{If:} \quad F(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}}) = 0$$

$$\quad \text{Then:} \quad F(\mathbf{a}, \mathbf{c}, \underbrace{\rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}}}_{\rho_{\mathbf{a},\mathbf{c}}}) \leq F(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$$

The FII_{comp} is only a *sufficient* condition for idempotency, not a necessary condition. Yet, for any faithfulness constraint which fails at the FII_{comp} , it is possible to construct an elementary counterexample where idempotency fails, as we will see below in section 5.

3. Second step: faithfulness constraints which (do not) satisfy the FII_{comp}

Proposition 1 provides a sufficient condition for idempotency in terms of the FII_{comp} stated in (9). This condition places no restrictions on the markedness constraints and instead only concerns the faithfulness constraints. The theory of idempotency thus turns into an investigation of the formal underpinning of theories of faithfulness: which faithfulness constraints satisfy the FII_{comp} ? This question is addressed by the following proposition, for a variety of faithfulness constraints within McCarthy & Prince’s (1995) Correspondence Theory and its extensions (references are only provided for constraints which were not included in McCarthy and Prince’s initial list).

Proposition 2 *Assume the candidate set (1) satisfies the reflexivity and transitivity axioms (2) and (8).*

- (a) *The following faithfulness constraints satisfy the FII_{comp} under no additional assumptions: segmental MAX, featural $\text{MAX}_{[\pm\varphi]}$ (Casali 1998), INTEGRITY.*
- (b) *The following faithfulness constraints satisfy the FII_{comp} under the assumption that no correspondence relation in the candidate set breaks any underlying segment: IDENT_{φ} (corresponding to a total feature φ), the local disjunction of any two identity faithfulness constraints (Downing 2000), segmental DEP, featural $\text{DEP}_{[\mp\varphi]}$ (Casali 1998), UNIFORMITY, LINEARITY, MAX/DEP-LINEARITY (Heinz 2005), ADJACENCY (Carpenter 2002).*
- (c) *The following faithfulness constraints fail at the FII_{comp} even when the correspondence relations in the candidate set are all one-to-one (no breaking nor coalescence): ANCHOR, CONTIGUITY, the local conjunction of any two conjoinable faithfulness constraints (Smolensky 1995). \square*

Note that McCarthy & Prince’s (1995) CONTIGUITY constraints and Carpenter’s (2002) ADJACENCY constraints are closely related and meant to serve the same purpose. Interestingly, only the latter satisfies the FII_{comp} . This shows that small differences in the definition of the constraints can have substantial formal consequences. These consequences are investigated in Magri (2015a:sections 4 and 6), which offers a proof of proposition 2. The rest of this section gives an informal idea of the reasoning.

Proposition 2a lists faithfulness constraints which satisfy the FII_{comp} without additional assumptions on the correspondence relations. To illustrate the reasoning, consider the constraint MAX. If the antecedent of the FII_{comp} (9) is false, the implication trivially holds. Thus, let’s suppose that the

of the FII_{comp} holds: the candidate $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b}, \mathbf{c}})$ in (12a) does not violate the conjoined constraint, because $/e/$ and $[i]$ only differ for the feature [high]. The right-hand side of the FII_{comp} inequality is small, namely equal to zero: the candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a}, \mathbf{b}})$ in (12c) does not violate the conjoined constraint, because $/a/$ and $[e]$ only differ for the feature [low]. The FII_{comp} inequality fails because its left-hand side is large, namely equal to 1: the composition candidate $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a}, \mathbf{b}}, \rho_{\mathbf{b}, \mathbf{c}})$ in (12b) violates the conjoined constraint, because $/a/$ and $[i]$ differ for both features [low] and [high].

$$(12) \quad \begin{array}{l} \text{a. } ID_{[\text{low}]} \wedge ID_{[\text{hi}]}(\mathbf{b}, \mathbf{c}) = 0 \\ \mathbf{b} = e \\ \quad | \\ \mathbf{c} = i \end{array} \quad \begin{array}{l} \text{b. } ID_{[\text{low}]} \wedge ID_{[\text{hi}]}(\mathbf{a}, \mathbf{c}) = 1 \\ \mathbf{a} = a \\ \quad | \\ \mathbf{c} = i \end{array} \quad \begin{array}{l} \text{c. } ID_{[\text{low}]} \wedge ID_{[\text{hi}]}(\mathbf{a}, \mathbf{b}) = 0 \\ \mathbf{a} = a \\ \quad | \\ \mathbf{b} = e \end{array}$$

4. Third step: extension to restricted constraints

A *restriction* R pairs a string \mathbf{a} with a subset $R(\mathbf{a})$ of its segments. A segment of the string \mathbf{a} *satisfies* the restriction provided it belongs to $R(\mathbf{a})$. The restriction R could look at segment position: $R(\mathbf{a})$ could be the set of the segments of \mathbf{a} which occupy its first syllable. Or it could look at segment quality: $R(\mathbf{a})$ could be the set of the consonants of \mathbf{a} . Or it could look at both: $R(\mathbf{a})$ could be the set of the consonants of \mathbf{a} which precede a sonorant according to \mathbf{a} . The faithfulness constraint MAX_R assigns to a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a}, \mathbf{b}})$ one violation for each segment of the underlying string \mathbf{a} which satisfies the restriction R and is deleted. Deletion of underlying segments which do not satisfy the restriction is not penalized. To illustrate, consider the restriction R which pairs a string \mathbf{a} with the set $R(\mathbf{a})$ of its consonants. The corresponding constraint MAX_R is the constraint $MAX-C$ which militates against consonant deletion, but is not offended by vowel deletion. The faithfulness constraint DEP^R assigns to a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a}, \mathbf{b}})$ one violation for each segment of the surface string \mathbf{b} which satisfies the restriction R and is epenthetic. To illustrate, consider the restriction R which pairs a string with the set of its vowels. The corresponding constraint DEP^R is the constraint $DEP-V$ which militates against vowel epenthesis, but is not offended by consonant epenthesis. The faithfulness constraint $IDENT_{\varphi, R}$ (and $IDENT_{\varphi}^R$) corresponding to a total feature φ assigns to a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a}, \mathbf{b}})$ one violation for each corresponding pair $(a, b) \in \rho_{\mathbf{a}, \mathbf{b}}$ of segments which differ for the value of the feature φ and whose underlying segment a (whose surface segment b) satisfies the restriction R . To illustrate, consider the restriction R which pairs a string with the set of its nasal segments. The corresponding constraints $IDENT_{[\text{nasal}], R}$ and $IDENT_{[\text{nasal}]}^R$ are Pater's (1999) constraints $IDENTI \rightarrow O[+\text{nasal}]$ and $IDENTO \rightarrow I[+\text{nasal}]$ which punish de-nasalization and nasalization, respectively. Proposition 2 extends to restricted constraints as follows.

Proposition 3 *Assume the candidate set (1) satisfies the reflexivity and transitivity axioms (2) and (8). Assume furthermore that no correspondence relation can cross the restriction R , namely can put in correspondence a segment which satisfies the restriction with a segment which does not satisfy it. Then MAX_R satisfies the FII_{comp} (9). Finally, assume that no correspondence relations can break an underlying segment. Then DEP^R , $IDENT_{\varphi, R}$, and $IDENT_{\varphi}^R$ satisfy the FII_{comp} (9) as well. \square*

The counterexample (13) shows that the FII_{comp} can fail when correspondence relations can cross the restriction. Let the restriction R pair a string with the set of its consonants and consider the restricted constraint $MAX_R = MAX-C$. The antecedent of the FII_{comp} holds: the candidate $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b}, \mathbf{c}})$ in (13a) does not violate $MAX-C$, because the deleted segment is a vowel. The right-hand side of the FII_{comp} inequality is equal to zero: the candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a}, \mathbf{b}})$ in (13c) does not violate $MAX-C$, because it involves no deletion. The FII_{comp} inequality thus fails, because its left-hand side is instead equal to 1: the composition candidate $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a}, \mathbf{b}}, \rho_{\mathbf{b}, \mathbf{c}})$ in (13b) does violate $MAX-C$, because it deletes a consonant. Crucially, the correspondence relation $\rho_{\mathbf{a}, \mathbf{b}}$ in (13c) crosses the restriction R : the consonant $/s/$ (which satisfies the restriction) corresponds to the vowel $[e]$ (which does not satisfy the restriction).

$$(13) \quad \begin{array}{lll} \text{a. } \text{MAX}_R(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}}) = 0 & \text{b. } \text{MAX}_R(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}}) = 1 & \text{c. } \text{MAX}_R(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}}) = 0 \\ \mathbf{b} = \begin{array}{c} ee \\ | \\ \mathbf{c} = e \end{array} & \mathbf{a} = \begin{array}{c} es \\ | \\ \mathbf{c} = e \end{array} & \mathbf{a} = \begin{array}{c} es \\ || \\ \mathbf{b} = ee \end{array} \end{array}$$

Proposition 3 can be substantially strengthened by substantially weakening the no-crossing assumption. For instance, in the case of $\text{IDENT}_{\varphi,R}$, the no-crossing assumption can be weakened to the assumption that no correspondence relation can exit from the restriction R without changing the value of the feature φ , namely can put in correspondence an underlying segment which satisfies the restriction with a surface segment which does not and yet has the same value for the feature φ .

5. Implications of idempotency for the OT analysis of chain shifts

A grammar is idempotent provided it fails at the implication (3) for some candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$. This means that it maps \mathbf{a} to $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ but \mathbf{b} to a candidate different from $(\mathbf{b}, \mathbf{b}, \mathbb{I}_{\mathbf{b},\mathbf{b}})$, say $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$. In other words, it displays the *chain shift* $\mathbf{a} \rightarrow \mathbf{b} \rightarrow \mathbf{c}$ (Łubowicz 2011; Moreton 2004a; Moreton & Smolensky 2002). This section uses the formal analysis of idempotency developed above to systematize various approaches to chain shifts which have been developed within *classic* OT, namely within the version of OT which assumes that the constraint set only consists of *markedness* constraints (which are blind to underlying forms) and *faithfulness* constraints (which assign no violations to the identity candidates).

5.1. Only a sufficient condition?

Consider an arbitrary (reflexive and transitive) candidate set, an arbitrary constraint set, and an arbitrary constraint ranking. Section 2 has established that the FII_{comp} is a *sufficient* condition for the idempotency of the corresponding OT grammar. This statement contains three universal quantifications: over candidate sets, over constraint sets, and over rankings. At this level of generality, the FII_{comp} is not only a sufficient but also a *necessary* condition for idempotency, in the following sense. Consider a faithfulness constraint F which does not satisfy the FII_{comp} (9). This means that there exist two candidates $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ and $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ such that: F assigns no violations to the candidate $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$, so that the antecedent of the FII_{comp} holds; but F assigns more violations to the candidate $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}})$ than to the candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$, so that the consequent fails. Suppose that the constraint set also contains a markedness constraint M which assigns more violations to the surface form \mathbf{b} than to the surface form \mathbf{c} .⁴ The OT grammar corresponding to the ranking $F \gg M$ then displays the chain shift $\mathbf{a} \rightarrow \mathbf{b} \rightarrow \mathbf{c}$ and thus fails at idempotency: the string \mathbf{b} is phonotactically licit, because the underlying form \mathbf{a} is mapped to the candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$, as shown in (14a); yet, the string \mathbf{b} does not surface faithfully, because the underlying form \mathbf{b} is not mapped to the identity candidate $(\mathbf{b}, \mathbf{b}, \mathbb{I}_{\mathbf{b},\mathbf{b}})$, as shown in (14b).

(14) a.	b.																		
<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding: 5px;">\mathbf{a}</td> <td style="padding: 5px;">F</td> <td style="padding: 5px;">M</td> </tr> <tr> <td style="padding: 5px;">$\mathbb{I}_{\mathbf{a},\mathbf{a}}(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$</td> <td style="padding: 5px;">*...*</td> <td style="padding: 5px;">*</td> </tr> <tr> <td style="padding: 5px;">$(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}})$</td> <td style="padding: 5px;">*...*!*</td> <td style="padding: 5px;"></td> </tr> </table>	\mathbf{a}	F	M	$\mathbb{I}_{\mathbf{a},\mathbf{a}}(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$	*...*	*	$(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}})$	*...*!*		<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding: 5px;">\mathbf{b}</td> <td style="padding: 5px;">F</td> <td style="padding: 5px;">M</td> </tr> <tr> <td style="padding: 5px;">$(\mathbf{b}, \mathbf{b}, \mathbb{I}_{\mathbf{b},\mathbf{b}})$</td> <td style="padding: 5px;"></td> <td style="padding: 5px;">*</td> </tr> <tr> <td style="padding: 5px;">$\mathbb{I}_{\mathbf{b},\mathbf{b}}(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$</td> <td style="padding: 5px;"></td> <td style="padding: 5px;"></td> </tr> </table>	\mathbf{b}	F	M	$(\mathbf{b}, \mathbf{b}, \mathbb{I}_{\mathbf{b},\mathbf{b}})$		*	$\mathbb{I}_{\mathbf{b},\mathbf{b}}(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$		
\mathbf{a}	F	M																	
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The rest of this section shows how various approaches to chain shifts in the classical OT literature fit within the schema (14), where F is one of the faithfulness constraints which were shown in sections 3 and 4 to fail at the FII_{comp} .

⁴ Furthermore, assume that there exists a markedness constraint (either M or a different markedness constraint ranked above M) which assigns more violations to the surface form \mathbf{a} than to the surface form \mathbf{b} . The latter markedness constraint is responsible for ruling out the candidate $(\mathbf{a}, \mathbf{a}, \mathbb{I}_{\mathbf{a},\mathbf{a}})$, which is therefore ignored in the rest of this section.

5.2. Chain-shifts through constraint conjunction

As noted in section 3, constraint conjunction yields faithfulness constraints which fail at the FII_{comp} . The use of constraint conjunction to model chain shifts within classical OT has been pioneered by Kirchner (1996) and systematized by Moreton & Smolensky (2002). To illustrate, consider the chain shift in the Lena dialect of Spanish illustrated in (15) (data from Hualde 1989 and Gnanadesikan 1997:section 5.4.3): because of the high final vowel in the masculine form, the underlying low and mid vowels (as revealed by the feminine form) raise to mid and high vowels, respectively.

- (15) $/a/ \rightarrow [e]$: *gáta* ‘cat-FEM’ *gétu* ‘cat-MAS’
 sánta ‘saint-FEM’ *séntu* ‘saint-MAS’
 $/e/ \rightarrow [i]$: *néna* ‘child-FEM’ *nínu* ‘child-MAS’
 séks ‘dry-FEM’ *síku* ‘dry-MAS’

The markedness constraint RAISE favors higher vowels before a high vowel. The conjunction of the two faithfulness constraints $IDENT_{[\text{high}]}$ and $IDENT_{[\text{low}]}$ punishes underlying low vowels mapped to surface high vowels. The analysis (16) is an instance of the scheme (14), with the conjoined constraint playing the role of the non- FII_{comp} faithfulness constraint.

(16)

<i>/gátu/</i>	$ID_{[\text{high}]} \wedge ID_{[\text{low}]}$ RAISE	
☞ <i>[gétu]</i>		*
<i>[gítu]</i>	*!	

<i>/nénu/</i>	$ID_{[\text{high}]} \wedge ID_{[\text{low}]}$ RAISE	
<i>[nénu]</i>		*!
☞ <i>[nítu]</i>		

Other approaches proposed in the literature are equivalent to the approach based on the conjunction of identity faithfulness constraints (see also the discussion of V-HEIGHTDISTANCE in Kirchner 1995). For instance, Gnanadesikan (1997:chapter 3) accounts for the chain shift $p \rightarrow b \rightarrow m$ (in post-nasal position) through the faithfulness constraint IDENT-ADJ, which is violated by a voiceless obstruent and a corresponding sonorant because they are separated by a distance larger than 2 on the *inherent voicing scale*. This constraint is thus equivalent to the conjunction $IDENT_{[\text{voice}]} \wedge IDENT_{[\text{son}]}$. Analogously, Dinnsen & Barlow (1998) account for the chain shift $s \rightarrow \theta \rightarrow f$ through the faithfulness constraint DISTFAITH, which is violated when the underlying and surface form differ by more than 1 on the scale $f=1$, $\theta=2$, and $s=3$ and is thus equivalent to the conjunction $IDENT_{[\text{coronal}]} \wedge IDENT_{[\text{strident}]}$. Yet, the approach based on constraint conjunction is more general than the latter approaches based on “scales”, as only the former extends to chain shifts which involve deletion (Moreton & Smolensky 2002). To illustrate, consider the Sea Dayak chain shift in (17) (data from Kenstowicz & Kisseberth 1979).

- (17) $/ŋgal/ \rightarrow [ŋa]$: */naŋgal/* *[nãŋaʔ]* ‘set up a ladder’
 $/ŋa/ \rightarrow [ŋã]$: */naŋa/* *[nãŋãʔ]* ‘straighten’

The analysis based on constraint conjunction extends as in (16) (based on Łubowicz 2011), which is another instance of the scheme (14).

(18)

<i>/naŋgal/</i>	$ID_{[\text{nas}]} \wedge \text{MAX}$	*NV
☞ <i>[nãŋaʔ]</i>		*
<i>[nãŋãʔ]</i>	*!	

<i>/naŋa/</i>	$ID_{[\text{nas}]} \wedge \text{MAX}$	*NV
<i>[nãŋaʔ]</i>		*!
☞ <i>[nãŋãʔ]</i>		

5.3. Chain shifts through constraint restrictions

As noted in section 4, constraint restriction yields faithfulness constraints which fail at the FII_{comp} when the correspondence relations are allowed to cross the restriction. This observation systematizes various approaches to chain shift proposed in the classical OT literature. For instance, Orgun (1995) considers the chain shift in (19) from Bedouin Hijazi Arabic: /a/ is raised to [i] but /i/ is deleted (both processes are restricted to short vowels in non-final open syllables; the data come from McCarthy 1993).

- (19) /a/ → [i]: /katab/ [k^hitab] ‘he wrote’
 /rafaagahl/ [ri^hfaagah] ‘companions’
 /i/ → ∅: /ʕarif+at/ [ʕarfat] ‘she knew’
 /ki^hill/ [k^hitil] ‘he was killed’

Orgun’s analysis is summarized in (20) — plus a markedness constraint [*a] which is omitted here. It relies on his constraint $\text{CORRESPOND}(/a/)$, which mandates that “every input /a/ has an output correspondent”. This constraint can be re-interpreted as MAX_R where the restriction R pairs a string with the set of its a’s. As shown in section 4, MAX_R fails at the FII_{comp} when correspondence relations are allowed to cross the restriction R , namely to put in correspondence a segment which satisfies the restriction with a segment which does not. That is precisely the case in (20), as the underlying /a/ (which satisfies the restriction R) corresponds to surface [i] (which does not satisfy the restriction). Orgun’s analysis (20) is thus an instance of the scheme (14), with the restricted constraint MAX_R playing the role of the non- FII_{comp} faithfulness constraint.

(20)

/a/	$\text{CORR}(/a/)$	*V
☞ [i]		*
∅	*!	

/i/	$\text{CORR}(/a/)$	*V
[i]		*!
☞ ∅		

As another example, Jesney (2007) considers the classical child chain shift in (21): coronal stridents are realized as coronal stops across the board; but coronal stops are velarized when followed by a lateral (data from Amahl age 2;2-2;11 as described in Smith 1973).

- (21) /s, z, ʃ, ʒ, tʃ, dʒ/ → [t, d]: [pʌdɔl] ‘puzzle’ [pʌtli:] ‘parsley’
 [pɛtɔl] ‘special’ [ændɔlɔ] ‘Angela’
 /t, d, n/ → [k, g, ŋ]: [pʌgɔl] ‘puddle’ [bʌklɔ] ‘butler’
 [tæŋgɔl] ‘sandal’ [bɔkɔl] ‘bottle’

Jesney’s analysis is summarized in (22). It relies on her “specific” faithfulness constraint $\text{IDENTCOR}/[+\text{strident}]$, which mandates that input stridents preserve their coronality. This constraint can be re-interpreted as $\text{IDENT}_{\varphi,R}$ where φ is the feature [coronal] and the restriction R pairs a string with the set of its stridents. As shown in subsection 4, $\text{IDENT}_{\varphi,R}$ fails at the FII_{comp} when correspondence relations are allowed to cross the restriction R . That is precisely the case in (22), as the underlying coronal /s/ (which satisfies the restriction R) corresponds to the surface coronal [t] (which does not satisfy the restriction). Jesney’s analysis (22) is thus an instance of the scheme (14), with the restricted constraint $\text{IDENT}_{[\text{cor}],R}$ playing the role of the non- FII_{comp} faithfulness constraint.

(22)

/s/	$\text{IDCOR}/[+\text{strid}]$	*TL
☞ [t]		*
[k]	*!	

/t/	$\text{IDCOR}/[+\text{strid}]$	*TL
[t]		*!
☞ [k]		

Note finally that, if only coronals can be [+strident], while all non-coronals are [–strident], then Jesney’s constraint IDENTCORONAL/[+strident] is equivalent to the conjunction $\text{IDENT}_{[\text{strident}]} \wedge \text{IDENT}_{[\text{coronal}]}$.

5.4. Chain-shifts through breaking

Let me close this section by discussing a fictional example. Kubozono et al. (2008) report that English *frog* is imported as [fu.róg.gu] into Japanese: the velar stop geminates (despite being voiced) because of a requirement on the placement of stress, captured here through a place-holder constraint STRESS. Assume an analysis of consonant gemination in terms of breaking: a single underlying consonant is put in correspondence with two identical surface consonants. Section 3 has shown that plain identity faithfulness constraints fail at the FII_{comp} when the correspondence relations are allowed to break underlying segments. This fact could be used to derive a fictional chain shift such as $\eta \rightarrow g \rightarrow gg$ through the analysis (23), which is an instance of the scheme (14) with the identity constraint playing the role of the non-FII_{comp} faithfulness constraint.

(23)

/ɨ/	IDENT _[nas]	STRESS
☞ [g]	*	*
[gg]	**	

/g/	IDENT _[nas]	STRESS
[g]		*
☞ [gg]		

6. Conclusion

A grammar is idempotent provided it faithfully maps any phonotactically licit phonological form to itself. Equivalently, a grammar fails at idempotency provided it displays at least one chain shift. Within constraint-based phonology, the typology of grammars is defined through a candidate set and a constraint set. Formal grammatical conditions such as idempotency must therefore be derivable from assumptions on the candidate and constraint sets, to the effect of excluding potentially dangerous candidates and constraints. This paper has pursued this idea within the (classical) OT implementation of constraint-based phonology. Building on Tesar’s (2013) theory of output-drivenness, idempotency has been reduced to a general condition on the faithfulness constraints (proposition 1). This condition has then been investigated for a number of faithfulness constraints which naturally arise within McCarthy & Prince’s (1995) Correspondence Theory, under various assumptions on the correspondence relations in the candidate set (propositions 2 and 3). The formal theory of idempotency thus developed has been used to systematize various approaches to chain shifts developed within the classical OT literature.

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