Quantification, Witness Sets and Conservativity

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1. Introduction

Theories of semantic scope underspecification can be divided into two types, depending on whether they are based on the notion of satisfaction or on the notion of derivation.[1] The central tenet of satisfaction-based theories of scope underspecification is that the relation between a representation SR with fully specified scope relations and a representation UR with underspecified scope relations is one of satisfaction: SR is a possible specification of underspecified representation UR iff SR satisfies UR.[2] On the other hand, the central idea of derivation-based theories of underspecification is that the relation between a fully specified representation SR and an underspecified representation UR is one of derivation: SR is a possible specification of an underspecified representation UR iff SR can be derived from UR.[3] What both types of theories have in common is that ultimately the denotation $[\![\text{UR}]\!]$ of an underspecified representation UR is the set of denotations $\{[\![\text{SR}_1]\!], \ldots, [\![\text{SR}_n]\!]\}$ of those fully specified representations $\text{SR}_1, \ldots, \text{SR}_n$ which (depending on the type of underspecification theory) either (i) satisfy UR or (ii) can be derived from UR.

According to this perspective on underspecification, then, an underspecified representation denotes a set of interpretations (readings), while the (partial or full) specification of an underspecified denotation amounts to narrowing down the set of interpretations. This perspective on underspecification has, arguably, two shortcomings. First, since the denotation of the underspecified representation is a set of fully specified readings, it does not capture what the different readings have in common. That is, this type of denotation for underspecified representations does not capture the fact that all readings involve the same verb denotation, and the same assignment of semantic roles to the determiner phrase denotations. And secondly, the denotation of the underspecified representation is not in any sense part of the denotation of each specified representation; on the contrary, there is a clear sense in which the denotations of the specified representations are part of the denotation of the underspecified representation, being elements of the underspecified denotation.

In this paper I will propose an alternative theory of quantification and scope underspecification, where (i) the denotation of the underspecified representation does indeed capture what the possible readings all have in common, and (ii) the specification of an underspecified denotation amounts to adding information, so that there is a clear sense in which the underspecified denotation is part of every specified reading.

The outline of the paper is as follows. In section 2 I provide an alternative to the Generalized Quantifier analysis of determiner phrase denotations, by claiming that DPs denote pairs consisting of the restrictor and the set of witnesses. To illustrate the basic idea, the sets of witnesses for $[\![$*every man*$]\!]$, $[\![$*no man*$]\!]$ and $[\![$*at most one man*$]\!]$ are $\{[\![man]\!]\}$, $\{\varnothing\}$ and $\{X : X \subseteq [\![man]\!] \wedge |X| \leq 1\}$, respectively, so

*I would like to thank the audience at the WCCFL29 conference as well as Marcus Kracht for valuable comments.

[1] See Egg (2011) for a survey of theories of underspecification.

[2] For example, in Hole Semantics introduced in Bos (1996) an object-level formula SR (e.g. a formula of Predicate Logic) is a possible specification of an underspecified meta-level representation UR (consisting of a set of metavariables called holes, a set of labeled formulas possibly containing holes, and a set of constraints on the relation between holes and labeled formulas) iff the bijective mapping between holes and labels that characterizes the object-level formula SR satisfies the constraints of the underspecified representation UR.

[3] For example, Ambiguous Predicate Logic (APL) introduced in Jaspars & van Eijck (1996) allows for formulas of Predicate Logic to be prefixed with a structured list of scope-bearing operators, which essentially represent a partial ordering on the scope-bearing operators. Further, APL defines a rewrite relation on formulas, such that SR is a possible specification of an underspecified representation UR iff it is possible to rewrite UR into SR by means of the specified rewrite relation.

that $[\![$*at most one man*$]\!] = \langle [\![man]\!], \{X : X \subseteq [\![man]\!] \wedge |X| \leq 1\}\rangle$. An important consequence of the fact that DP denotations are no longer analyzed as functions from a (scope) set into truth-values is that the number of possible DP denotations is considerably reduced. Assuming that the denotation type fixes the search space for the learner, the reduction of the number of possible DP denotations is attractive, as it narrows down the search space and thus simplifies the learnability problem (other things being equal). The semantic composition rules combining verb (phrase) and determiner phrase denotations, presented in section 3, assign semantic roles to each DP denotation, and record this assignment in a so-called store, so that e.g. the sentence *More than half the referees read at least three papers.* denotes the pair $\langle [\![read]\!], \{\langle 1, [\![$*more than half the referees*$]\!]\rangle, \langle 2, [\![$*at least three papers*$]\!]\rangle\}\rangle$ consisting in the verb denotation $[\![read]\!]$ and the store $\{\langle 1, [\![$*more than half the referees*$]\!]\rangle, \langle 2, [\![$*at least three papers*$]\!]\rangle\}$. Importantly, this underspecified clause denotation captures what all possible readings have in common (namely the verb denotation itself and the assignment of semantic roles), while leaving the scope dependencies unspecified. Section 4 introduces the intuitive idea behind the sequential expansion of relations. Continuing the illustration, the expansion of the second projection adds the pair $\langle x, [\![$*at least three papers*$]\!]\rangle$ to the verb denotation $[\![read]\!]$ for every individual x, if (i) x stands in the reading relation to a set of papers P, (ii) the store contains the pair $\langle 2, [\![$*at least three papers*$]\!]\rangle$, and (iii) the set P is a witness of $[\![$*at least three papers*$]\!]$. Expanding the first projection next, the pair $\langle [\![$*more than half the referees*$]\!], [\![$*at least three papers*$]\!]\rangle$ is added if the set of individuals standing in the reading relation to at least three papers is a witness of $[\![$*more than half the referees*$]\!]$. If this is the case, the sentence is true in the given model relative to this order of expansion, otherwise false. Expanding the second projection and then the first projection yields the reading where the object has narrow scope, whereas expanding the first projection and then the second projection corresponds to the reading where the object has wide scope. In order to analyse cumulative readings I introduce simultaneous expansion in addition to sequential expansion. Section 5 provides a formalisation of these ideas, and section 6 concludes the paper.

2. Restricting determiner denotation type

In Generalized Quantifier Theory, determiners like for example *every*, *most*, *no* denote binary functions from subsets of the domain to truth-values (or equivalently relations between subsets of the domain). Determiner phrases like *every man*, *most girls*, *no student* denote unary functions from subsets of the domain to truth-values. Here we shall depart from this analysis, and propose instead that (i) determiners are unary functions, taking as their single argument the restrictor set, and (ii) determiner phrases refer to pairs $\langle R, W\rangle$ consisting of a restrictor set R, and the set W of subsets of R, called witness sets. To illustrate, let the domain be $D = \{p_1, p_2, p_3, p_4, p_5, r_1, r_2, r_3, r_4\}$, and let the denotations of *paper* and *referees* be $[\![paper(s)]\!] = \{p_1, p_2, p_3, p_4, p_5\}$ and $[\![referees]\!] = \{r_1, r_2, r_3, r_4\}$, respectively.

$$[\![\textit{every}]\!](A) := \langle A, \{X : X \subseteq A \wedge X = A\}\rangle = \langle A, \{A\}\rangle$$
$$[\![\textit{every}]\!]([\![\textit{referee}]\!]) = \langle \{r_1, r_2, r_3, r_4\}, \{\{r_1, r_2, r_3, r_4\}\}\rangle$$

$$[\![\textit{no}]\!](A) := \langle A, \{X : X \subseteq A \wedge X = \varnothing\}\rangle = \langle A, \{\varnothing\}\rangle$$
$$[\![\textit{no}]\!]([\![\textit{paper}]\!]) = \langle \{p_1, p_2, p_3, p_4, p_5\}, \{\varnothing\}\rangle$$

$$[\![\textit{more than half the}]\!](A) := \langle A, \{X : X \subseteq A \wedge |A - X| < |A \cap X|\}\rangle$$
$$[\![\textit{more than half the}]\!](\{r_1, r_2, r_3, r_4\}) = \langle \{r_1, r_2, r_3, r_4\}, \{\{r_1, r_2, r_3\}, \{r_1, r_2, r_4\}, \{r_1, r_3, r_4\}, \{r_2, r_3, r_4\}, \{r_1, r_2, r_3, r_4\}\}\rangle$$

Given a determiner phrase *DP* with $[\![$*DP*$]\!] = \langle R, W\rangle$, the negation $[\![$*not DP*$]\!]$ consists of the same restrictor set R and the set of subsets of R which are not in W, i.e. $\langle R, \wp(R) - W\rangle$, which shall be represented as $\langle R, \overline{W}\rangle$. To illustrate again:

$$
\begin{array}{lcl}
[\![\textit{not every}]\!](A) & := & \langle A, \{X : X \subseteq A \wedge X \neq A\}\rangle \\
[\![\textit{not every}]\!]([\![\textit{referee}]\!]) & = & \langle \{r_1, r_2, r_3, r_4\}, \{\varnothing, \{r_1\}, \{r_2\}, \{r_3\}, \{r_4\}, \{r_1, r_2\}, \{r_1, r_3\}, \\
 & & \{r_1, r_4\}, \{r_2, r_3\}, \{r_2, r_4\}, \{r_3, r_4\}, \{r_1, r_2, r_3\}, \{r_1, r_2, r_4\}, \\
 & & \{r_1, r_3, r_4\}, \{r_2, r_3, r_4\}\}\rangle
\end{array}
$$

$$
\begin{array}{lcl}
[\![\textit{not exactly two}]\!](A) & := & \langle A, \{X : X \subseteq A \wedge |X| \neq 2\}\rangle \\
[\![\textit{not exactly two}]\!]([\![\textit{referees}]\!]) & = & \langle \{r_1, r_2, r_3, r_4\}, \{\emptyset, \{r_1\}, \{r_2\}, \{r_3\}, \{r_4\}, \{r_1, r_2, r_3\}, \\
 & & \{r_1, r_2, r_4\}, \{r_1, r_3, r_4\}, \{r_2, r_3, r_4\}, \{r_1, r_2, r_3, r_4\}\}\rangle
\end{array}
$$

The conjunction of two DP denotations $\langle R, W\rangle$ and $\langle R', W'\rangle$ is the pair consisting of the union $R \cup R'$ of the two restrictor sets, and the set of pairwise unions of witness sets in W and W', i.e. $\langle R, W\rangle \wedge \langle R', W'\rangle = \langle R \cup R', \{w \cup w' : w \in W \wedge w' \in W'\}\rangle$. For example, the denotation of *every referee and at most one paper* is:

$$
\begin{array}{lcl}
[\![\textit{every referee and at most one paper}]\!] & = & [\![\textit{every referee}]\!] \wedge [\![\textit{at most one paper}]\!] \\
 & = & \langle [\![\textit{paper}]\!], \{\emptyset, \{p_1\}, \{p_2\}, \{p_3\}, \{p_4\}, \{p_5\}\}\rangle \\
 & = & \langle [\![referee]\!] \cup [\![paper]\!], \\
 & & \{\{r_1, r_2, r_3, r_4\}, \{r_1, r_2, r_3, r_4, p_1\}, \{r_1, r_2, r_3, r_4, p_2\}, \\
 & & \{r_1, r_2, r_3, r_4, p_3\}, \{r_1, r_2, r_3, r_4, p_4\}, \\
 & & \{r_1, r_2, r_3, r_4, p_5\}\}\rangle
\end{array}
$$

The exclusive disjunction of two DP denotations $\langle R, W\rangle$ and $\langle R', W'\rangle$ is the pair consisting of the union $R \cup R'$ of the two restrictor sets, and the union of the sets W and W' of witness sets, i.e. $\langle R, W\rangle \wedge \langle R', W'\rangle = \langle R \cup R', W \cup W'\rangle$. The denotation of *every referee or at most one paper* is:

$$
\begin{array}{lcl}
[\![\textit{every referee or at most one paper}]\!] & = & [\![\textit{every referee}]\!] \vee [\![\textit{at most one paper}]\!] \\
 & = & \langle [\![\textit{referee}]\!], \{r_1, r_2, r_3, r_4\}\rangle \vee \\
 & & \langle [\![\textit{paper}]\!], \{\emptyset, \{p_1\}, \{p_2\}, \{p_3\}, \{p_4\}, \{p_5\}\}\rangle \\
 & = & \langle [\![referee]\!] \cup [\![paper]\!], \\
 & & \{\{r_1, r_2, r_3, r_4\}, \emptyset, \{p_1\}, \{p_2\}, \{p_3\}, \{p_4\}, \{p_5\}\}\rangle
\end{array}
$$

I close this subsection with two remarks and an important consequence. The first remark is that since determiner phrases refer to a pair of sets, the present theory of quantifying expressions should be classified as a referential rather than a quantificational theory of quantifying expressions. The second remark is that unlike in Generalised Quantifier Theory, the semantics of the determiner does not make reference to the scope set but only to the restrictor set. This has an important consequence, namely that the number of possible determiner denotations is considerably reduced, compared to Generalised Quantifier Theory. Given a universe of discourse D containing exactly two entities and the set $T = \{0, 1\}$ of truth-values, there are four subsets of D, $2^4 = 16$ functions from subsets of D into T (i.e. 16 possible determiner phrase denotations), and $2^{16} = 65536$ functions from subsets of D into DP denotations (i.e. 65536 possible determiner denotations). Out of these only 512 determiner denotations are conservative, where a determiner denotation D is conservative iff $D(A)(B) \leftrightarrow D(A)(A \cap B)$. To illustrate, the determiner denotation $[\![most]\!]$ is a conservative function, since the sentence *Most students smoke* is true (in a model M) if and only if *Most students are students who smoke.* is true (in M). Based on the fact that no clear example of a determiner DET has yet been found, where the equivalence $[\![DET]\!](A)(B) \leftrightarrow D(A)(A \cap B)$ does not hold, Barwise & Cooper (1981) and Keenan & Stavi (1986) have hypothesized that all natural language determiner denotations are conservative.

If, on the other hand, determiners are functions from subsets R of D into pairs $\langle R, W\rangle$, where W is a set of subsets of R, i.e. $W \subseteq \wp(R)$, then there are exactly 512 possible determiner denotations. To see this, note that given the same universe of discourse $D = \{a, b\}$ there are $n_1 \times n_2 \times n_4 \times n_4$ possible determiner denotations, where $n_1 = 2$ is the number of possible witnesses given the restrictor set $\varnothing$ (the first witness is $\varnothing$, and the second is $\{\varnothing\}$), $n_2 = 4$ is the number of possible witnesses given the restrictor set $\{a\}$ (namely $\varnothing$, $\{\varnothing\}$, $\{\{a\}\}$ and $\{\varnothing, \{a\}\}$), $n_3 = 4$ is the number of possible witnesses given the restrictor set $\{b\}$ (namely $\varnothing$, $\{\varnothing\}$, $\{\{b\}\}$ and $\{\varnothing, \{b\}\}$), and $n_4 = 16$ is the number of possible witnesses given the restrictor set $\{a, b\}$ (namely all the subsets of $\{\varnothing, \{a\}, \{b\}, \{a, b\}\}$. So there are

$2 \times 4 \times 4 \times 16 = 512$ possible DET denotations, showing that the present proposal considerably reduces the class of possible DET denotations. Assuming that the denotation type fixes the search space for the learner, the reduction of the class of possible DP denotations is attractive, as it narrows down the search space and thus simplifies the learnability problem (other things being equal).

3. Reformulating verb denotation and semantic composition

In what follows we shall separate the analysis of semantic role assignment from the analysis of the resolution of scope dependencies. The main motivation for this is the observation that in many languages the assignment of semantic roles is (i) encoded grammatically and therefore (ii) independent of the context of use, whereas the resolution of scope dependencies is often (i) not encoded grammatically, and moreover (ii) dependent on the particular lexical items involved as well as the particular context of use.[4] Semantic role assignment will be achieved by SEMANTIC COMPOSITION RULES, whereas scope dependencies are determined by the way in which EXPANSION RULES apply.

The semantic composition rules combining DP and verb (phrase) denotations assign a semantic role of the verb (phrase) denotation to the DP denotation. Given a verb like *give*, we shall assume that it assigns three (generalised) semantic roles, which we simply name $1, 2$ and 3, so that the person being assigned the role 1 gives the entity assigned the role 2 to the person assigned the role 3. If a DP denotation $[\![DP]\!]$ is assigned the (generalised) semantic role 2, we shall record this information by adding the pair $\langle 2, [\![DP]\!]\rangle$ to the verb (phrase) denotation. This requires the verb (phrase) denotation to be analyzed as a pair consisting of a relation R and a store S. To illustrate, consider:

(1) *More than half the referees read at least three papers.*

The denotation of this clause can be computed in two different ways. One possibility is to combine the verb and direct object denotations first, and then combine the resulting denotation with the subject denotation.

$$\mathcal{O}_2(\langle [\![\textit{read}]\!], \emptyset\rangle, [\![\textit{at least three papers}]\!]) = \langle [\![\textit{read}]\!], \{\langle 2, [\![\textit{at least three papers}]\!]\rangle\}\rangle$$

$$\mathcal{O}_1(\langle [\![read]\!], \{\langle 2, [\![at\ least\ three\ papers]\!]\rangle\}\rangle, [\![more\ than\ half\ the\ referees]\!]) =$$
$$\langle [\![read]\!], \{\langle 1, [\![more\ than\ half\ the\ referees]\!]\rangle, \langle 2, [\![at\ least\ three\ papers]\!]\rangle\}\rangle$$

The second possibility is to combine the subject with the verb first, and then combine the resulting denotation with the object denotation:

$$\mathcal{O}_1(\langle [\![\textit{read}]\!], \emptyset\rangle, [\![\textit{more than half the referees}]\!]) = \langle [\![\textit{read}]\!], \{\langle 1, [\![\textit{more than half the referees}]\!]\rangle\}\rangle$$

$$\mathcal{O}_2(\langle [\![\textit{read}]\!], \{\langle 1, [\![\textit{more than half the referees}]\!]\rangle\}\rangle, [\![\textit{at least three papers}]\!]) =$$
$$\langle [\![\textit{read}]\!], \{\langle 1, [\![\textit{more than half the referees}]\!]\rangle, \langle 2, [\![\textit{at least three papers}]\!]\rangle\}\rangle$$

Given a (finite) set S of semantic roles (which, for convenience, are represented by means of integers), the SEMANTIC COMPOSITION RULE (schema) is defined as follows:

$$\mathcal{O}_i([\![DP]\!], \langle R, S\rangle) = \langle R, S \cup \{\langle i, [\![DP]\!]\rangle\}\rangle$$

for every semantic role $i \in$ S.

Given these semantic composition rules, the DP denotations can combine with the verb denotation in any order, without having to type-lift either DP or verb denotation (as required in type-driven approaches to semantic composition). Semantic composition theories that are equally flexible without requiring type-shifting include among others the recent approaches in Beaver & Condoravi (2007) and Eckardt (2010).

[4]Despite this observation, there are a number of theories of quantification which do not separate semantic role assignment from the resolution of scope dependencies. For example, in Heim & Kratzer (1998) both the semantic role of a DP denotation as well as its scope is fixed at the same point in the derivation, namely when the DP denotation is combined with a property.

4. Expansion

4.1. Sequential expansion - the basic idea

To complete our toy model, assume that

$$\begin{aligned}[\![\textit{read}]\!] = \{ \; & \langle r_1, p_1\rangle, \langle r_1, p_2\rangle, \langle r_1, p_3\rangle, \langle r_1, p_5\rangle, \langle r_1, b_1\rangle, \langle r_1, b_2\rangle, \\ & \langle r_2, p_1\rangle, \langle r_2, p_2\rangle, \langle r_2, p_4\rangle, \langle r_2, b_2\rangle, \\ & \langle r_3, p_2\rangle, \langle r_3, p_3\rangle, \langle r_3, p_4\rangle, \langle r_3, b_1\rangle, \\ & \langle r_4, p_4\rangle, \langle r_4, b_1\rangle, \langle r_4, b_2\rangle, \\ & \langle s_1, p_1\rangle\}\end{aligned}$$

which, for ease of readability, will be represented as:

$$\begin{array}{lll} r_1 & \rightarrow & p_1, p_2, p_3, p_5, b_1, b_2 \\ r_2 & \rightarrow & p_1, p_2, p_4, b_2 \\ r_3 & \rightarrow & p_2, p_3, p_4, b_1 \\ r_4 & \rightarrow & p_4, b_1, b_2 \\ s_1 & \rightarrow & p_1 \end{array} \quad \text{or} \quad \begin{array}{lll} r_1, r_2, s_1 & \rightarrow & p_1 \\ r_1, r_2, r_3 & \rightarrow & p_2 \\ r_1, r_3 & \rightarrow & p_3 \\ r_2, r_3, r_4 & \rightarrow & p_4 \\ r_1 & \rightarrow & p_5 \\ r_1, r_3, r_4 & \rightarrow & b_1 \\ r_1, r_2, r_4 & \rightarrow & b_2 \end{array}$$

So in this toy model, the sentence (2) denotes (3):

(2) *More than half the referees read at least three papers.*

(3) $\langle [\![\textit{read}]\!], \{\langle 1, [\![\textit{more than half the referees}]\!]\rangle, \langle 2, [\![\textit{at least three papers}]\!]\rangle\}\rangle$

(3) is an underspecified denotation which (i) expresses what all the possible specified readings have in common, while (ii) leaving the scope dependencies unspecified. The scope dependencies are specified by applying expansion operations to this underspecified denotation. The idea behind the sequential expansion operation, which was initially inspired by Akiba (2009), is to add tuples to the relation R of a clause denotation $\langle R, S\rangle$. To illustrate, the expansion of the second projection of the relation $[\![\textit{read}]\!]$ in (3) adds the pair $\langle x, [\![\textit{at least three papers}]\!]\rangle$ to the verb denotation $[\![read]\!]$ for every individual x which stands in the reading relation to a set of papers P, if (i) the store contains the pair $\langle 2, [\![\textit{at least three papers}]\!]\rangle$, and (ii) the set P is a witness of $[\![\textit{at least three papers}]\!]$. In our model, r_1 read the papers p_1, p_2, p_3, p_5. Since $\{p_1, p_2, p_3, p_5\}$ is a witness set of $[\![at\ least\ three\ papers]\!]$ the expansion of the second projection adds $\langle r_1, [\![\textit{at least three papers}]\!]\rangle$ to $[\![\textit{read}]\!]$. Secondly, r_2 read the papers p_1, p_2, p_4, and since $\{p_1, p_2, p_4\}$ is a witness set of $[\![at\ least\ three\ papers]\!]$, the expansion of the second projection adds $\langle r_2, [\![\textit{at least three papers}]\!]\rangle$ to $[\![\textit{read}]\!]$. Finally, r_3 read the papers p_2, p_3, p_4, and since $\{p_2, p_3, p_4\}$ is a witness set of $[\![at\ least\ three\ papers]\!]$ the operation expanding the second projection adds $\langle r_3, [\![\textit{at least three papers}]\!]\rangle$ to $[\![\textit{read}]\!]$. Consequently, the result of expanding the second projection of $[\![\textit{read}]\!]$ in (3) is $\langle X, \{\langle 1, [\![more\ than\ half\ the\ referees]\!]\rangle, \langle 2, [\![at\ least\ three\ papers]\!]\rangle\}\rangle$ where

$$X = \begin{array}{lll} r_1 & \rightarrow & p_1, p_2, p_3, p_5, b_1, b_2 \\ r_2 & \rightarrow & p_1, p_2, p_4, b_2 \\ r_3 & \rightarrow & p_2, p_3, p_4, b_1 \\ r_4 & \rightarrow & p_4, b_1, b_2 \\ s_1 & \rightarrow & p_1 \\ r_1 & \rightarrow & [\![\textit{at least three papers}]\!] \\ r_2 & \rightarrow & [\![\textit{at least three papers}]\!] \\ r_3 & \rightarrow & [\![\textit{at least three papers}]\!] \end{array}$$

Next, the first projection of X is expanded. Since (i) in X the only referees who read at least three papers are r_1, r_2, r_3, (ii) the pair $\langle 1, [\![\textit{more than half the referees}]\!]\rangle$ belongs to the store, and (iii) the set r_1, r_2, r_3 is a witness set of the subject DP denotation $[\![\textit{more than half the referees}]\!]$, the expansion of the first projection of X adds the pair $\langle [\![\textit{more than half the referees}]\!], [\![\textit{at least three papers}]\!]\rangle$, so that the resulting (fully specified) denotation is $\langle Y, \{\langle 1, [\![more\ than\ half\ the\ referees]\!]\rangle, \langle 2, [\![at\ least\ three\ papers]\!]\rangle\}\rangle$ where:

$$
\begin{array}{lcll}
Y & = & r_1 & \rightarrow & p_1, p_2, p_3, p_5, b_1, b_2 \\
 & & r_2 & \rightarrow & p_1, p_2, p_4, b_2 \\
 & & r_3 & \rightarrow & p_2, p_3, p_4, b_1 \\
 & & r_4 & \rightarrow & p_4, b_1, b_2 \\
 & & s_1 & \rightarrow & p_1 \\
 & & r_1 & \rightarrow & [\![\textit{at least three papers}]\!] \\
 & & r_2 & \rightarrow & [\![\textit{at least three papers}]\!] \\
 & & r_3 & \rightarrow & [\![\textit{at least three papers}]\!] \\
 & & [\![\textit{more than half the referees}]\!] & \rightarrow & [\![\textit{at least three papers}]\!]
\end{array}
$$

4.2. Truth relative to expansion

Given the denotation (3) of the sentence (2) we were able to expand the second projection of $[\![\textit{read}]\!]$ and then the first projection of X such that the pair $\langle [\![\textit{more than half the referees}]\!], [\![\textit{at least three papers}]\!] \rangle$ has been added to the relation. In this case we shall say that the sentence is true in the given model M relative to expanding the second projection and then the first projection, i.e. under the reading where the subject has wide scope.

However, if we expand the first projection before the second projection, this pair cannot be added, as shown in the next subsection, and therefore the sentence is false in this model relative to the inverse order of expansion.

4.3. Inverse scope reading

The evaluation of the inverse reading begins with the expansion of the first projection of $[\![\textit{read}]\!]$. To see more clearly which entities stands in the reading relation to a given entity, we use the following representation of $[\![read]\!]$:

$$
\begin{array}{lcl}
r_1, r_2, s_1 & \rightarrow & p_1 \\
r_1, r_2, r_3 & \rightarrow & p_2 \\
r_1, r_3 & \rightarrow & p_3 \\
r_2, r_3, r_4 & \rightarrow & p_4 \\
r_1 & \rightarrow & p_5 \\
r_1, r_3, r_4, s_1 & \rightarrow & b_1 \\
r_1, r_2, r_4, r_2 & \rightarrow & b_2
\end{array}
$$

As can be seen, the paper p_1 is read by exactly two referees, namely r_1 and r_2. Since in our model $\{r_1, r_2\}$ is not a witness of $[\![\textit{more than half the referees}]\!]$, the expansion of the first projection cannot add the pair $\langle [\![\textit{more than half the referees}]\!], p_1 \rangle$ to $[\![\textit{read}]\!]$. However, the paper p_2 is read by the referees r_1, r_2 and r_3, and since (in our model) $\{r_1, r_2, r_3\}$ is a witness of $[\![\textit{more than half the referees}]\!]$, the expansion of the first projection adds the pair $\langle [\![\textit{more than half the referees}]\!], p_2 \rangle$ to $[\![\textit{read}]\!]$. Further, p_4, b_1 and b_2 are read by $\{r_2, r_3, r_4\}$, $\{r_1, r_3, r_4\}$ and $\{r_1, r_2, r_4\}$ respectively. Since these sets are witnesses of $[\![\textit{more than half the referees}]\!]$, the three pairs $\langle [\![\textit{more than half the referees}]\!], p_4 \rangle$, $\langle [\![\textit{more than half the referees}]\!], b_1 \rangle$ and $\langle [\![\textit{more than half the referees}]\!], b_2 \rangle$ are added to $[\![\textit{read}]\!]$, resulting in:

$$
\begin{array}{lcl}
r_1, r_2, s_1 & \rightarrow & p_1 \\
r_1, r_2, r_3 & \rightarrow & p_2 \\
r_1, r_3 & \rightarrow & p_3 \\
r_2, r_3, r_4 & \rightarrow & p_4 \\
r_1 & \rightarrow & p_5 \\
r_1, r_3, r_4, s_1 & \rightarrow & b_1 \\
r_1, r_2, r_4, r_2 & \rightarrow & b_2 \\
[\![\textit{more than half the referees}]\!] & \rightarrow & p_2 \\
[\![\textit{more than half the referees}]\!] & \rightarrow & p_4 \\
[\![\textit{more than half the referees}]\!] & \rightarrow & b_1 \\
[\![\textit{more than half the referees}]\!] & \rightarrow & b_2
\end{array}
$$

As can now be seen, if the first projection is expanded first, there are only two papers which are being read by ⟦*more than half the referees*⟧, namely p_2 and p_4. But the set $\{p_2, p_4\}$ is not a witness of ⟦*at least three papers*⟧, and therefore the expansion of the second projection cannot add the pair ⟨⟦*more than half the referees*⟧, ⟦*at least three papers*⟧⟩. This shows that the sentence (2) is not true in this model relative to the inverse scope reading (analyzed as expanding the first projection of ⟦*read*⟧ before the second projection).

4.4. Downward entailing DP denotations

Consider next the following sentence containing the downward entailing expression *no referee*.[5]

(4) *No referee read every paper.*

Note that in our toy model this sentence is true under the direct scope reading. For this to come out right, the expansion of the second projection before the first projection should result in the addition of the pair ⟨⟦*no referee*⟧, ⟦*every paper*⟧⟩ to ⟦*read*⟧. However, since in our model no individual stands in the reading relation to every paper, the expansion strategy employed so far does not add any pair containing ⟦*every paper*⟧ as its second element. What is missing is that the expansion operator also adds negative information, i.e. information about what is not the case. We shall deal with this in two steps. First, if ⟦*every paper*⟧ is assigned the semantic role 2, and an individual x stands in the reading relation to a set P of papers which is not a witness of ⟦*every paper*⟧, we shall let (sequential) expansion add the pair ⟨x, ⟦*not every paper*⟧⟩, resulting in:

r_1	$\rightarrow$	$p_1, p_2, p_3, p_5, b_1, b_2$
r_2	$\rightarrow$	p_1, p_2, p_4, b_2
r_3	$\rightarrow$	p_2, p_3, p_4, b_1
r_4	$\rightarrow$	p_4, b_1, b_2
s_1	$\rightarrow$	p_1
r_1	$\rightarrow$	⟨⟦*not every paper*⟧⟩
r_2	$\rightarrow$	⟨⟦*not every paper*⟧⟩
r_3	$\rightarrow$	⟨⟦*not every paper*⟧⟩
r_4	$\rightarrow$	⟨⟦*not every paper*⟧⟩
s_1	$\rightarrow$	⟨⟦*not every paper*⟧⟩

If every referee stands in the reading relation to ⟦*not every paper*⟧, that is just another way of saying that no referee read every paper. Therefore, the second step in dealing with negative information in our example is to let the expansion of the first projection add the pair ⟨⟦*no referee*⟧, ⟦*every paper*⟧⟩, provided that the following condition on the expansion of the first projection is met: (i) the set of referees standing in the reading relation to ⟦*not every paper*⟧ is not a witness set of ⟦*no referee*⟧, (ii) no pair ⟨x, ⟦*every paper*⟧⟩ with x a referee can be added, and (iii) the semantic role 1 is assigned to a downward entailing DP denotation. Since this condition is indeed satisfied, expansion of the first projection results in:

r_1	$\rightarrow$	$p_1, p_2, p_3, p_5, b_1, b_2$
r_2	$\rightarrow$	p_1, p_2, p_4, b_2
r_3	$\rightarrow$	p_2, p_3, p_4, b_1
r_4	$\rightarrow$	p_4, b_1, b_2
s_1	$\rightarrow$	p_1
r_1	$\rightarrow$	⟦*not every paper*⟧
r_2	$\rightarrow$	⟦*not every paper*⟧
r_3	$\rightarrow$	⟦*not every paper*⟧
r_4	$\rightarrow$	⟦*not every paper*⟧
s_1	$\rightarrow$	⟦*not every paper*⟧
⟦*no referee*⟧	$\rightarrow$	⟦ *every paper*⟧

[5] Within the present theory, a DP denotation is downward entailing iff the set of witnesses contains $\varnothing$.

showing that the sentence (4) is true in our model under the direct scope reading.[6]

4.5. Cumulative readings

So far expansion applied sequentially to each projection of an n-ary relation. In order to account for cumulative readings another expansion operation is required which applies to the projections simultaneously. To illustrate, consider our sentence (5) in the new model (6):

(5) *More than half the referees read at least three papers.*

(6)
$$\begin{array}{lcl} [\![referee]\!] & = & \{r_1, r_2, r_3, r_4\} \\ [\![paper]\!] & = & \{p_1, p_2, p_3, p_4, p_5\} \\ [\![student]\!] & = & \{s_1, s_2, s_3\} \\ [\![book]\!] & = & \{b_1, b_2\} \end{array} \qquad [\![read]\!] = \begin{array}{ll} r_1 & \rightarrow p_1, b_1 \\ r_2 & \rightarrow p_2, b_2 \\ r_3 & \rightarrow p_3, b_1, b_2 \\ r_4 & \rightarrow b_1, b_2 \\ s_1 & \rightarrow p_1, b_1 \end{array}$$

(5) is false in this new model under both the direct and the inverse scope interpretation. None of the referees reads papers which form a witness set of ⟦*at least three papers*⟧, so expansion of the second projection cannot add any pair with ⟦*at least three papers*⟧ as the second element, and therefore the pair ⟨⟦*more than half the referees*⟧, ⟦*at least three papers*⟧⟩ cannot be added by expansion of the first projection either. Expanding the first projection before the second projection is equally unsuccessful, because no paper has been read by more than half the referees, showing that the sentence is false under the inverse scope reading, too. However, $\{r_1, r_2, r_3\}$ is a witness set of ⟦*more than half the referees*⟧ and $\{p_1, p_2, p_3\}$ is a witness sets of ⟦*at least three papers*⟧. In this case, simultaneous expansion is defined such that ⟨⟦*more than half the referees*⟧, ⟦*at least three papers*⟧⟩ gets added to the relation ⟦*read*⟧.

5. Formalizing expansion and truth

Let $\langle V, S\rangle$ be a clause denotation, with V an n-ary relation and $\langle i, \langle R, W\rangle\rangle \in S$. To (sequentially) i-expand V at the i-th position, do the following for every n-ary sequence σ (so also for $\sigma \notin V$):

1. find all sequences $\tau \in V$ which differ from σ at most in the entity occurring in the i-th position. If τ differs from σ at most in the i-th element, we write $\tau \sim_i \sigma$
2. build the set of elements σ_i^V which are in the i-th position of one such $\tau \in V$, and then intersect σ_i^V with the restrictor set R, where $\sigma_i^V := \{\pi_i(\tau) : \tau \sim_i \sigma \wedge \tau \in V\}$
3. if $\sigma_i^V \cap R$ is a witness set, then add $\sigma[i/\langle R, W\rangle]$ to V, where $\sigma[i/\langle R, W\rangle]$ is the result of replacing the i-th element of σ by $\langle R, W\rangle$
4. if $\sigma_i^V \cap R$ is not a witness set, then add $\sigma[i/\langle R, \overline{W}\rangle]$ to V, where $\overline{W} = \{X : X \subseteq R \wedge X \notin W\}$
5. if (i) $\langle j, \langle R_j, W_j\rangle\rangle \in S$ for some projection $j \neq i$, (ii) there is a $\tau \in V'$ with $\pi_j(\tau) = \langle R_j, \overline{W_j}\rangle$, (iii) there is no $\rho \in V'$ with $\pi_j(\rho) = \langle R_j, W_j\rangle$, (iv) $\varnothing \in W_i$ (restriction to downward entailing DP denotations), and (v) $\tau_i^V \cap R$ is not a witness set of W_i, then $\tau[i/\langle R_i, W_i\rangle, j/\langle R_j, W_j\rangle\rangle] \in V'$.

(7) **Definition (sequential i-expansion):**
Let $[\![\phi]\!] = \langle V, \{\langle i, \langle R_i, W_i\rangle\rangle : 1 \leq i \leq n\}\rangle$ be the denotation of an n-ary formula ϕ. Then the **sequential i-expansion EXP$_i^{SEQ}$**$([\![\phi]\!])$ of $[\![\phi]\!]$ is the pair $\langle V', S\rangle$ where V' is the smallest set satisfying the following conditions:

1.$V \subseteq V'$

2.for all n-ary sequences σ:

•$\sigma_i^V \cap R_i \in W_i \rightarrow \sigma[i/\langle R_i, W_i\rangle] \in V'$
•$\sigma_i^V \cap R_i \notin W_i \rightarrow \sigma[i/\langle R_i, \overline{W_i}\rangle] \in V'$

[6] See section 5 for an explicit formalisation of the sequential i-expansion operation.

3.for all $j \neq i$ with $\langle j, \langle R_j, W_j \rangle\rangle \in S$ such that there is a $\tau \in V'$ with $\pi_j(\tau) = \langle R_j, \overline{W_j}\rangle$, and there is no $\rho \in V'$ with $\pi_j(\rho) = \langle R_j, W_j\rangle$

$\bullet(\varnothing \in W_i \wedge \tau_i^V \cap R \notin W_i) \to \tau[i/\langle R_i, W_i\rangle, j/\langle R_j, W_j\rangle\rangle] \in V'$.

(8) **Definition (simultaneous expansion)**:
Let $[\![\phi]\!] = \langle V, S\rangle$ be the denotation of an n-ary formula with $\langle j, \langle R_j, W_j\rangle \in S$ for all $j, 1 \leq j \leq n$. Then $\text{EXP}_i^{SIM}([\![\phi]\!]) = \langle V', S\rangle$, where V' is the smallest set satisfying the following conditions:

1.$V \subseteq V'$

2.if $\exists X_1, \ldots, X_n.(X_1 \in W_1 \wedge \ldots \wedge X_n \in W_n \wedge$
$X_1 = \{\pi_1(\sigma) : \sigma \in V\} \wedge \ldots \wedge X_n = \{\pi_n(\sigma) : \sigma \in V\})$
then $\langle X_1, \ldots, X_n\rangle \in V'$

(9) **Definition (truth relative to sequence of expansions)**:
An n-ary formula ϕ with $[\![\phi]\!] = \langle V, \{\langle i, X_i\rangle : 1 \leq i \leq n\}\rangle$ is **true relative to a sequence of expansions** $\langle \alpha_1, \ldots, \alpha_m\rangle, 0 \leq m$ iff

$$\langle X_1, \ldots, X_n\rangle \in \pi_1(\alpha_1(\alpha_2(\ldots \alpha_m([\![\phi]\!]))))$$

6. Conclusion

In this paper I sketched (the beginnings of) a theory of quantification and scope underspecification, where (i) the denotation of the underspecified representation does indeed capture what the possible readings all have in common, and (ii) the specification of an underspecified denotation amounts to adding information, so that there is a clear sense in which the underspecified denotation is part of every specified reading. Unlike in GQT, conservativity follows (i) from the restriction of the denotation type of determiners, and (ii) from the specification of semantic composition and expansion rules. Unlike e.g. in Hendriks (1993), the combination of verb and DP denotations does not require (i) the DP denotations to combine in a certain order, (ii) the scope dependencies to be fixed beforehand, and (iii) the verb denotation to be lifted/expanded to the worst case (in order to anticipate all possible DP denotations). Unlike in Cooper (1983), the result of combining DP and verb denotations is not the set of all possible readings, but a relation-store pair capturing what all possible expansions have in common, namely the verb denotation and the assignment of semantic roles to DP denotations.

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Proceedings of the 29th West Coast Conference on Formal Linguistics

edited by Jaehoon Choi, E. Alan Hogue, Jeffrey Punske, Deniz Tat, Jessamyn Schertz, and Alex Trueman

Cascadilla Proceedings Project Somerville, MA 2012

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ISBN 978-1-57473-451-5 library binding

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This paper can be cited as:

Klein, Udo. 2012. Quantification, Witness Sets and Conservativity. In *Proceedings of the 29th West Coast Conference on Formal Linguistics*, ed. Jaehoon Choi et al., 125-133. Somerville, MA: Cascadilla Proceedings Project. www.lingref.com, document #2695.